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2 A Global Pulpwood Supply Model and Some Implications

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A GLOBAL PULPWOOD SUPPLY MODEL AND SOME IMPLICATIONS

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Background

This study involves the development of a global pulpwood supply model (PSM) through the process of modifying our earlier Timber Supply Model (TSM). The TSM is fully developed in our book *The Long-Term Adequacy of Global Timber Supply*, by Sedjo and Lyon (1990), published by Resources for the Future. The TSM uses an economic supply/demand approach to project an inter-temporal time path of the world's price and output level of industrial wood. Additionally, the model provides projections of the time path of the equilibrium output levels of the various regions into which the world has been subdivided.

Overview of the Model

The purpose of the model is to function as a tool to assist in the task of assessing of the condition and the adequacy of the long-term world timber supply. The model is a useful vehicle for systematizing and formalizing the factors that affect long-term supply as well as examining the nature of the forces and the interrelationships within and among supply regions. In addition, the projections can identify questionable logical implications of the assumptions of the model and/or the specific conditions associated with the projections.

The ability of models to examine the implications of alternative assumptions and situations is one of their major strengths. A formal model allows the user to examine possible futures under various assumptions regarding relationships and events. Since the assumptions must be explicit, it forces the analyst to define precisely the assumptions and to confront the implications of the assumptions. Fundamental deficiencies or logical inconsistencies in a model's structure and/or underlying assumptions, are reflected in the form of implausible projections. These types of results forces the analyst to reconsider the structure and assumptions.

For example, the PSM demonstrates that if pulpwood demand grows sufficiently more rapidly than overall industrial wood demand, an implication is that there will be an absolute decline in the production and consumption of solidwood, as pulpwood bids raw wood away from solidwood uses. This finding reveals the interrelationship of demand for pulpwood and demand for industrial wood. They are not wholly independent. However, some analysts make the mistake of trying to assess the global pulpwood market without recognizing its relationship with total wood supply.

A working hypothesis of this study is that, in the aggregate, timber production in the real world is experiencing a transition from the draw down of existing old growth stands to the utilization of second-growth and plantation-grown industrial wood. This transition is at different stages in different regions of the globe. A basic question that the model is designed to address is that of determining the economically optimal transition from and old growth forest to a regulated steady-state forest.[1] This approach is in conflict with the common "growth/drain" approach to modelling forest harvest.

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[1] Foresters are fond of the regulated or steady-state forest in which there is a) an equal area of forest land for each age class; b) a fixed rotation age; and c) an age class for each year to rotation. Under this condition each year's harvest will be the same and steady-state production will be achieved.

In the growth/drain approach supply is made a function of the stock of forest inventory. Because the growth-drain approach has no provision for the age distribution of the inventory, harvests are invariant to the age composition of the forest. While such approaches are useful when applied to even-aged regulated forests, such an approach can lead to serious errors when applied to a non even-aged situation. In fact, there is no mechanism for a transition from an old-growth to an even-aged regulated forest and no movement toward a long-term equilibrium. In a world in which the transition from old-growth to steady-state harvests is not completed, such an approach is not appropriate beyond the individual forest stand.

The TSM utilizes a control theory approach that introduces "initial conditions" and "laws of motion" for the forest system. "Control" variables are introduced to monitor and describe the changing "state" or condition of the forest. The initial conditions refer to conditions that obtain initially, such as the forest inventory by location, age group and land class. Since the initial conditions include old growth and various other non regulated timber stands, the approach requires "laws of motion," rules that govern the system over time, that can address an initial stock that includes large volumes of old-growth.

In the control theory approach, the changing age and volume conditions of the forest are constantly monitored and updated so that management decisions explicitly recognize the changing state of the forest. In this approach the laws of motion have young trees becoming older and as older trees are harvested; either natural regeneration occurs on the site or investments in regeneration are made. Any investments, in turn, influence the rate of growth of the forest. The control variables, or choice variables, give the area harvested by age group and land class for each year. This, in turn, determines the rotation age by land class. Also control variables determine the investment in regeneration and the magnitude of regeneration input each year by land class.

An optimization procedure calculates the values of the control and state variables in the steady state. A solution algorithm then solves for the optimal values of the control variables in the transition between the initial conditions and the conditions of the steady state. The optimal control variable values generate an evolution of the state variables from their initial values to their steady state values that identify the economically optimal time path of price and harvest between the initial conditions and the terminal steady state. The economically optimal time path is the one that maximizes the sum of consumer and producer's surpluses in the transition between the initial conditions and the steady state.

Rational Expectations

The model is a rational expectations model in that the solution assumes that economic agents *on the average* correctly anticipate future conditions.[2] One implication of this formulation is that if future demand is going to increase substantially, economic agents adjust to the anticipate higher future prices by reducing harvests today to be able to increase harvests in the future. This behavior results in shifting harvests from the present to the future. Lower current harvests result in increased current prices, while increased future harvests decrease future prices. Similarly, the expectation of higher future prices will result in increased investments in regeneration today.

[2] Robert Lucas recently received the Nobel prize in economics for developing the rational expectations approach. Many timber models are not rational expectations models. For example, the US Forest Service's TAMM solves each period independently. In this model, economic agents are assumed to have no expectations of even the next year's price and harvest conditions.

Factors Affecting Harvest Levels

In the TSM harvest levels can be affected by adjustments of six types:

1. the rate of draw down of old growth inventories;
2. the number and size of forest land classes utilized for harvest;
3. the rotation length;
4. the level of regeneration inputs, which influences future harvest levels;
5. the rate of technological change in timber growing; and
6. the rate at which industrial plantations are added to the world's timber producing regions.

The first three of these relate to the rate at which existing forests are harvested and are determined within the model (endogenously) and are affected by the current and future prices, as well as the interest rate. These can be viewed as short- or medium-term effects. The last three relate to the rate at which new sources of wood are made available or used more efficiently.

The fourth of these output adjustments, investments in regeneration, is determined endogenously in the model. This activity influences future harvest levels. The fifth is dependent on the rate at which technological progress is incorporated into the yield function and thereby effects timber growth. Its effect on timber growth enters the system via the regeneration component in the supply model. The sixth effect is the rate at which new industrial plantations are established. In the timber supply model the base case assumed that 200,000 ha of new plantation be established annually, beginning in year one for 30 years. This assumption is revised in the PSM.

Finally, technological change can also enter the TSM via the demand side.[3]

The Timber Supply Model (TSM)

The Pulpwood Supply Model (PSM)is derived from the broader Timber Supply Model. In this section we will first discuss some specific features of the TSM and then in later sections move on to a detailed discussion and application of the PSM.

On the supply side the TSM subdivides the world into 8 regions, seven of which were formally modeled and called "responsive regions."[4] The "rest of the world" was lumped

[3] A host of technological innovations have been wood saving and wood extending in that allow intermediate and final products to be produced utilizing less wood, or lower quality wood, than previously. Examples include wood saving pulping techniques and new types of engineered wood, such as oriented strand board (OSB). The effect of wood saving technology is to reduce the wood requirements of various intermediate and final products using wood thereby lowering the rate of outward shift in the demand curve for industrial wood for a given rate of increase in the demand for the final product.

[4] The industrial wood sectors of the seven "responsive regions" are treated as being driven market forces under competitive conditions. Although this is not completely correct, it is probably a good first approximation. Even where a large portion of the market is served by public forests, as was the case in the US before the 1990s and still, to a lesser extent, is the case today, the results can be viewed as

together as the eighth region and referred to as the "non-responsive region." The harvests of the non-responsive region are viewed as autonomous and determined independently of the usual economic considerations. This characterization is clearly only approximate in that some areas within this region do respond to market incentives. However, since this region includes the former centrally planned economies, many of which have not yet developed the institution of efficiently working markets, and also some regions of Europe, which are claimed to have a traditional of longer than financially optimum rotations, the characterization of non-responsive is probably a good overall approximation.

The seven responsive regions were further subdivided into a total of 22 timber land classes each of which corresponds to a unique area. The regions for the PSM are the same. The regions are:

Responsive Regions
- Emerging Region[5] (1 land class)
- US Pacific Northwest (4 land classes)
- Canada, west (2 land classes)
- Canada, east (4 land classes)
- US South (8 land classes)
- Nordic Region (2 land classes)
- Asia-Pacific (1 land class)

Non-responsive Region
- Rest of the World

The model incorporates physical and biological elements to provide a nature system framework or what economists might call and underlying biological production function. For each of the 22 land classes the model incorporates physical and biological information to develop a production function. This includes information on land class quality, location, accessibility, and area; growth and yield function by dominant species and land class; existing inventories and their age distribution; suitability of timber for sawlogs or pulping; and silvicultural responses to investment inputs.[6] The amount of investment in forest regeneration and management is determined endogenously.[7] In some situations a land class is not harvested for certain time periods because either there is no mature timber or because the stumpage price does not justify harvesting this land class, given the harvest and transport costs and the alternative wood sources available.[8]

Supply side technological change is introduced into the model through the land class yield functions. To the extent that management is applied to the regenerated forest, it is

approximating the market since the large private sector responds to price signals generated in part by the harvest levels of the public sector. Thus, the market Timber Supply Model can be applied.

[5] The Emerging Region is a composite consisting of a number of regions that are producing industrial wood from intensively managed exotic species tree plantations. These include countries such as Brazil, Chile, Indonesia, New Zealand, South Africa and Spain. Although the species, growth rates and rotations vary somewhat across regions, all these plantations have relatively rapid growth and short rotations.

[6] Suitability is determined largely by average log size (width) with large logs having a greater fraction of their total volume going to solidwood.

[7] See appendix N of *The Long-Term Adequacy of World Timber Supply* (1990).

[8] This feature means that the timber base is allowed to expand and contract depending upon the endogenously determined price.

incorporated into the yield function of that year's regeneration cohort. The increased yield was then associated with that age class and is captured at harvest.[9]

Also, each land class has a unique set of costs including establishment, growing, harvesting, transport to a pulp mill and international transport costs. The industrial wood product was assumed to be processed in a local pulp mill and transported to the world market. Since the focus of this analysis is the resource, the mill costs were assumed to be identical across regions. The costs of transporting the processed product to the world market and the world market price jointly determined the net price the mill received for the processed product. The world market was treated as consisting of three sub markets - eastern North American, western and central Europe, and East Asia. Prices among these markets could differ, but not by more than transport costs, since arbitrage was assumed to limit the price differentials.

The level of output of the non-responsive region is viewed as independent of market conditions. This aggregate region was assumed to continue its production over the fifty year period with the growth of annual output based upon historic trends. Its production was assumed initially to be expanding at 0.5 percent annually, falling linearly to zero at the end of the 50 year period. In the period 1985-1995 total world industrial wood production was roughly divided equally between the seven responsive regions and the non-responsive region.

On the demand side total world industrial wood demand was first interacted with the known non-responsive region supply to generate an excess or derived demand curve for the industrial wood of the responsive region. In the TSM the total world industrial wood demand function was initially assumed to be shifting out a 1.0 percent annually[10] linearly declining to zero growth in year 50.[11] This, in turn, generates an initial rate of expansion of about 1.5 percent annually for the excess demand curve that is applied to the responsive region. The excess demand curve was then related to the supply conditions (production and cost functions) of the 22 land classes of the responsive regions to generate the supply curve for the responsive regions.

The Pulpwood Supply Model (PSM)

The PSM is derived from the TSM and the formal mathematics appear in Appendix A. As with the TSM, the PSM divides the world into eight supply regions, seven of which are "responsive" to market forces and are explicitly modeled, and one "non-responsive" region, which is defined as the "rest of the world." The responsive regions are further subdivided into 22 timber land cases, each with unique timber growing (production function) and locational characteristics. In the PSM each land class is also given an initial mix between pulpwood and solidwood uses.

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[9] We assume that technology is progressing at an initial rate of 0.5 percent annually decreasing linearly to zero in year 50. The rate is embodied in the current year's yield function and introduced through the regeneration input. If a forest is wholly naturally regenerated no technology is introduced. If there is $500 of regeneration per ha, the entire 0.5 percent is captured by that year's age class. Regeneration between zero and $500 is prorated proportionally. See chapter 6 and appendix L of *The Long-Term Adequacy* (1990).

[10] The rate of growth of world demand for industrial wood from 1970 to 1991 reported by the FAO was 1.0 percent. Also, no attempt is made in this model to forecast the business cycle and the projects should be interpreted as long-term trends.

[11] For tractability, the trends in the model converge to zero in year 50. Therefore the more useful projections occur in the early part of the period, roughly the first three decades.

Model Modifications

As noted, the PSM was developed by modifying the original TSM. These modifications consist of two basic types. First the TSM was updated to reflect current conditions. An adjustment is made to update the initial conditions of the TSM to reflect the situation applicable to the PSM. These modifications are:

i) Changes in timber inventory base used in the original TSM to reflect the passage of time and harvests which have occurred during the intervening period between the completion of the TSM several years ago and the current development of the PSM. The initial timber inventories were adjusted to reflect the 10 years difference in the beginning conditions, i.e., the TSM's initial year was 1985 while that for the PSM was 1995.[12]

ii) An adjustment of the land base to reflect changes in the timber available for harvest that have occurred as the result of policy. The forest inventory bases in the Pacific Northwest and British Columbia have been reduced by one-third and fifteen percent respectively.

iii) The initial conditions were adjusted to correct for the more rapid than anticipated establishment of plantations in the Emerging region. Thus the area in plantations and the vintage of the plantations was adjusted to reflect the higher establishment rates that occurred in the 1980s and early 1990s.[13]

iv) Adjustment of other initial conditions of the model to reflect the 1995 situation were undertaken. For example, although a precise comparison of the initial year of the projections of the PSM with the actual pulpwood production in 1993 (Wood Resources International Ltd. 1995) is difficult because the regional breakdowns in the data are somewhat different and the 1993 data are not strictly comparable to our 1995 initial year, the initial harvests of the PSM are nevertheless reasonably comparable to those which were estimated by WRI for that period (Table 1). This is, of course, partly due to crude calibrations of the PSM initial conditions to the WRI data of 1993.

v) An increase in the base case rate of plantation establishment reflecting increased actual establishment rates.[14] In addition, the average yield rates of the plantation region were adjusted[15] so as to more accurately reflect average performance and to calibrate the initial

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[12] The updating of inventories is achieved simply by running the model to the year desired and making some modest adjustments to provide crude compatibility with the results of Hagler's (1995) report. In the control theory process the model keeps track of the decreases in inventories that result from harvesting as well as the increases in inventories due to growth, management and new plantations.

[13] The base case of the TSM assumed 200,000 ha established annually. The revised model increases this to an average of about 500,000 ha established annually through the late 1980s and early 1990s, which appears to be the best estimate of plantation establishment in recent years.

[14] Our best estimates of current annual rates of industrial plantation establishment from nontraditional regions are 200,000 ha for South America, 150,000 ha for Southeast Asia including Indonesia, and about 150,000 ha for Oceania. In addition, China is undertaking tree planting at a substantial, but difficult to quantify rate, and other regions also have positive rate of plantation establishment. In the base case we assume that plantations in nontraditional regions will be added at 600,000 ha per year declining gradually to zero in year 50.

[15] To accomplish this the yield function of the emerging plantation region was reduced from an average of 20 cubic meters per ha per year to 17.5 cubic meters. This adjustment also reflects the well recognized phenomenon that the actual rates of growth in emerging region plantations, while high, are on the average somewhat lower than the better growth rates often cited. This reflects the fact the cited growth rates are commonly taken from experimental plots which typically have better than average growth rates.

base case plantation harvest levels of the PSM with the information on pulpwood supplies in 1993 as provided by Wood Resources International Ltd.[16]

vi) Also, some modest changes were made in the harvesting costs for land class 7, the more inaccessible areas of western Canada. Specifically, the harvest cost for the land class were reduced from $19.00 to $13.00 per cubic meter and the domestic transport costs from $12.35 to $9.00 per cubic meter to be more reflective of the actual role this region plays in production.[17]

Table 1. World Pulpwood Production in 1993 (million cubic meters)

Asia	38
Africa	11
Oceania	14
United States and Canada	355
Other, North America	1
South/Central America	39
Western Europe	119
Eastern Europe	9
Former Soviet Union	33
Total	618

Source: Wood Resources International Ltd. 1995, p. 8.

The second change from the conditions of the earlier TSM is to make appropriate modifications and disaggregations in elements of the TSM so as to generate the PSM. Two types of modifications of the TSM were taken to convert the model into the PSM. These are a) on the demand side, and b) on the supply side.

Demand

The demand function for the PSM expands on the single output of the TSM. The pulpwood demand is added to the model as a subset of industrial wood demand. Implicitly, the solidwood demand function is the residual of the industrial wood demand function less the pulpwood demand function. Thus, changes in solidwood demand are determined by the change of total industrial wood demand less the change of pulpwood demand.[18]

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[16] No attempt was made to precisely relate the 1995 figures of the PSM with the estimates for 1993 obtained by Wood Resource International (WRI). Beside the differences in the year, the figures reflect cyclical phenomena which is not intended to be captured in the PSM.

[17] The location of major pulp mills in parts of northern Canada and the introduction of activities with scale economics, has reduced harvest and transport costs for many locations.

[18] The implication of this structure is that, for any given rate of growth of total industrial wood demand, the more rapid the growth rate of pulpwood demand, the less rapid the growth of solidwood demand. A result of this formulation is that for a sufficiently rapidly growing pulpwood demand, solidwood demand would need to be declining. This is perhaps not as unlikely a real world event as it may seem. For example, during the period 1900-1985 total industrial wood growth in the US was only 0.81 percent annually.

On the demand side, the initial rate of growth in the industrial wood demand function was maintained in 1995 at the 1.0 percent annual increase rate that was applied the initial year of the TSM, 1985. This change reflects the reality that the actual growth of the industrial wood demand curve over the period 1985-1995 was somewhat more rapid than anticipated in the TSM.[19]

Based on recent output levels, an initial pulpwood demand growth of 2.25 percent.[20] In all cases the growth of the demand function was linearly reduced to zero by the end of the period. Under these conditions, over the period the pulpwood share of industrial wood demand is allow to grow from about 35 percent in 1995 to about 60 percent in 2045. This changing share reflects the anticipated long-term increase of pulpwood in total industrial wood demand.

Since the pulpwood demand function is a component of the total industrial wood demand function, the more rapid growth of the pulpwood demand function implies a reduced demand function growth rate for solidwood. This implication shows up strongly as slow and/or negative solidwood growth in many of our projections.

Supply

The PSM views pulpwood supply as coming from two sources. These are: a) timber harvests which are undertaken explicitly to generate pulpwood and b) as by-products of industrial solidwood production in the form of sawmill residues. The timber resources of each the 22 land classes are allocated between solidwood and pulpwood. The initial division is based on the nature of the forest, e.g., typical log size, species, usual rotation age. In the PSM solidwood and pulpwood can be substituted for one another, within limits.

In principle, all solidwood can be converted to pulpwood, but not all pulpwood can be converted to solidwood. For each land class an initial solidwood/pulpwood mix is given based on the nature of the timber in a land class. The actual proportions are allow to vary within a range on either side of the initial proportions depending upon the relative price of solidwood and pulpwood.[21]

With these modifications the TSM now has additional initial conditions. In addition to land area for each land class, inventory age and volume by age, and yield function by land class, it also has as part of the initial conditions the mix between pulpwood and solidwood, by land class, including provision for mill residues becoming pulpwood. Also, included is a substitution function whereby the mix between pulpwood and solidwood can change within some limits as a function of the relative prices of pulpwood and solidwood.

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[19] Data on world production of forest products is notoriously lagged in its compilation and release. In mid 1995 the latest data available by the FAO is for 1993. However, given the upturn in worldwide economic activity in late 1994 and 1995, it is likely that the 1995 data, when released, will reflect the current rising business cycle.

[20] Worldwide demand for pulpwood grew at an annual rate of 2.53 percent between 1964 and 1985. However, for the sub period 1970-85 worldwide growth rate was only 1.4 percent. The FAO world pulpwood growth reported for the most recent period, 1980-1991 was 1.8 percent annually.

[21] From the initial sawnwood/pulpwood proportion, the amount of solidwood can increase a maximum of 5%. However, pulpwood can increase to consume the entire log. The initial (reference) proportions and those of the base case and extreme high demand scenario are presented in Appendix B.

The Solution

The PSM is a sub-component of the TSM. The TSM is solved given the known initial conditions, which now include both initial total industrial wood demand and pulpwood demand levels, as well as the rates of change of these demands over the total period. The model is then solved for the steady state[22] solution of both outputs by land class and price. Next, one of the set of feasible time paths, e.g., that which traces the path from the initial conditions to the steady state, is identified. Finally the optimal time path, which maximizes the sum of producers and consumers surplus, was identified from among the feasible paths.

Since the pulpwood solution is generated within the overall context of the TSM, pulpwood production is constrained to be equal or be less than industrial wood production for each region. Solidwood production for each region is calculated as the difference between total industrial wood production and pulpwood production.

Base Case

The base case presented in this report is in many regards an extension of the base case of the TSM as it appeared in our 1990 book, *The Long-Term Adequacy of Global Timber Supply.* This is viewed by the authors as the most likely outcome and it is against this case that the various scenarios are compared. The assumptions used in the PSM for the base-case forecast are as follows:

1. World demand for industrial wood initially increases at an annual rate of 1.0 percent, falling linearly in successive years to zero after fifty years.

2. World demand for pulpwood initially increases at an annual rate of 2.25 percent, falling linearly in successive years to zero after fifty years.

3. The production of the non-responsive region increases at a rate of 0.5 percent annually, falling linearly to zero after fifty years.

4. Bio-technological change is assumed to shift the yield functions upward to a maximum of 0.5 percent annually, falling linearly to zero after fifty years. Technological change is introduced via investments in regeneration. The rate for any specific land class is a function of the amount of regeneration investment varying between a maximum of 0.5 percent for regeneration investments of $500 per ha or more, falling to no technological change for zero investment in regeneration.

5. New forest plantations are established in the emerging region at a annual level of 600,000 ha, falling linearly to zero at year 50.

6. The dollar exchange rate is assumed to remain at the current level throughout the period of analysis.[23]

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[22] The steady state solution is that equilibrium to which the global industrial system adjusts after which it provides a continuous given output over time.

[23] For a discussion of exchange rates used see *The Long Term Adequacy of World Timber Supply*, pages 204 and 205.

A Global Overview of the Base Case

Overall global pulpwood production (figure 1.1) increases from about 700 million cubic meters in 1995 to about 1.325 billion in 2045.[24] Thus there is almost a doubling of the production of pulpwood over the 50 year period. Although the almost doubling of output is not a huge increase for the five decade period, it does appear to be reasonable in the context of the expanding production from newly developed plantations over the period, the wood saving technological change that is occurring in pulping, such as new technologies that increase wood pulping yields, e.g., thermo-mechanical or groundwood pulping approaches, the modest increase in tree yields due to technology and the worldwide increase in the use of recycled paper as a substitute for virgin fiber.

Additional insights into the nature of the base case and indeed the PSM can be gained be viewing pulpwood production as a part of the larger global industrial wood production. Figure 1.3 presents the total world industrial wood base case production by the eight regions for the 50 year period 1995-2045. Over that period total industrial wood production increases from about 1.7 billion cubic meters to 2.3 billion cubic meters, an increase of about 35 percent over a five decade period. However, as noted in Figure 1.1, total pulpwood production essentially doubles over that period. This large shift in the composition of industrial wood production away from solidwood to pulpwood is necessary to accommodate the more rapidly rising demand for pulpwood. To accomplish this change in composition of output all or most the individual regions must change their mix of outputs away from solidwood and toward pulpwood. This shift also occurs in the non-responsive region.

The implication of the above is that, worldwide, total solidwood production must decline to allow such a large shift in the composition of production without even larger increases in total output. This, in fact, is projected on figure 1.4 as total world solidwood production falls from almost 1.1 billion cubic meters in 1995 to about 980 million in 2045.

Finally, figure 1.5 shows the real price trends of pulpwood and solidwood over the 50 year period. Pulpwood price shows a fairly substantial increase throughout the first one-third of the period, a more modest increase over the second third, and a slight decline during the last third. Solidwood prices are almost the inverse of pulpwood, declining over the first third of the decade, increasing slightly over the next third and increasing in the last third of the decade. Over the whole of the 50 year period overall price increases are rather modest being about 30 percent for pulpwood and only about 8 percent for solidwood. The rise in pulpwood prices in the early period reflects the rapid rate at which pulpwood demand is expanding relative to solidwood. Thus, the pulpwood price must increase to attract wood from solidwood to pulpwood uses.[25]

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[24] This increase is generated from both the responsive and non-responsive regions. The responsive region increases come as the result of increases in technology, management and new plantations, as well as the addition of increased harvests from the marginal land classes that, in the PSM, are induced into production by higher prices. The output from the non-responsive region increases on the basis of historical trend and may well involve inclusion of the harvests from additional forest lands.

[25] In the various scenarios examined only the base case and the integrated supply constrained have the very large difference between the rate of growth of pulpwood demand and that of total wood demand. This difference implies that solidwood demand is growing very slowly, if at all. In this case the price of pulpwood rapidly approaches that of solidwood as, through the early part of the period, the pulpwood market must bid wood away from solidwood market. A comparison of the prices of the base case and those of scenario 5 (Figure 6.5) is instructive.

The Responsive Regions

Figure 1.1 and figure 1.2 present the base case projections of total world pulpwood for the seven responsive regions and the aggregate non-responsive region. Total world pulpwood production is projected to increase from about 700 billion cubic meters in 1995 to about 1.325 billion cubic meters in 2045. The contribution of the various regions can be read in greater detail from figure 1.2, which focuses on the outputs of the seven responsive regions. The regions accounting for most of the present and projected incremental output are the Emerging region followed by the US South. Production of the emerging region grows to almost twice that of the South after 50 years. In addition eastern Canada is a substantial pulpwood producer throughout the fifty year period, with some increase in the last one and one-half decades. The PNW and western Canada remain significant but modest producers of pulpwood. Although experiencing some fluctuations over time in production, the Nordic region continues as a major producer of pulpwood throughout the period. Finally, throughout the entire period the Asia-Pacific region is only a very modest producer of pulpwood. The projections of the Asia-Pacific, however, do not incorporate the substantial expansions in pulpwood plantations and pulp mills that are envisaged in some Asia-Pacific countries.[26]

The dominant producing regions in 1995 are the Emerging region and the US South. The Emerging region experiences roughly a doubling of pulpwood harvests, while the South increases production roughly by about one-third over the 50 year period in the base case projection. In addition, the Nordic and Eastern Canada regions maintain their production over the 50 year period, with Eastern Canada showing an modest increase. The three regions with modest pulpwood production, the US Pacific Northwest, western Canada and the Asia-Pacific region, show only minimal harvest changes over the 50 years.

The base case PSM projections show the seven responsive regions increasing their pulpwood production from about 560 million cubic meters in 1995 to 875 million cubic meters in 2045, or from roughly 80 percent of 1995 world total pulpwood production about 66 percent of world projected pulpwood production in 2045. The extent of the decline in the portion of pulpwood provided by the seven responsive regions over the five decade period, however, may be in part an anomaly of the unique situation existing in the early 1990s and built to some degree into the initial conditions of the PSM. The 1993 data reflected the fact that sawnwood production in the former Soviet Union, was well below is historical trend, due to the disruptions generated by the political changes that occurred in the early 1990s. Thus the dramatic decline in the share of total pulpwood from the responsive regions reflects this abnormality of the early 1990s.

Scenarios

In this section we undertake a number of scenarios that explore the implications of hypothesized changes in the conditions applicable to the pulpwood industry. These include:

1. Decreasing Demand
2. Moderately High Demand (based on FAO forecasts)
3. Very High Demand
4. Integrated Supply Constraints with Base Case Demand
5. Integrated Supply Constraints with Low Demand
6. Very High Demand with High Plantation Establishment

[26] For example, some early Indonesia plans anticipated having some 32 pulpmills by the year 2020, compared to the 14 mills currently in operation. However, our projections assume plantations being established at about the current rate of 125,000 annually and are captured in the output of the emerging region.

Scenario 1. Decreasing Demand

This scenario examines the implications of an absolute decline in demand for pulpwood and industrial wood over the next 50 years.[27] Total industrial wood demand is posited to decline at an rate of 1.0 percent in the initial year, linearly moving to zero percent demand change in year 50. Pulpwood demand is posited to decline at a rate of 1.5 percent in the initial years linearly moving to a zero percent demand change in year 50.

The projections from the decreasing demand scenario appear in figures 2.1 though 2.5. World pulpwood production declines from about 620 million cubic meters in 1995 (year 0) to 570 million cubic meters in 2045 (year 50).[28] All of the responsive regions except the Emerging regions experience pulpwood output declines over the 50 years (figure 2.2). Eastern and western Canada as well as the Asia-Pacific produce essentially no pulpwood in year 50. The US South and the Nordic region both produce significant, but reduced, volumes in year 50. The Pacific Northwest pulpwood production is modest throughout the period, but lower in year 50.

Total worldwide industrial wood production (figure 2.3) also declines in this scenario, from about 1.7 billion cubic meters in 1995 to roughly 1.5 billion in 2045. Total solidwood production changes little over the 50 year period (figure 2.4), rising from 320 million to 360 million cubic meters over the 50 years, with the highest production level, roughly 400 million, being reached after about 30 years.

Finally, both solidwood and pulpwood prices decline throughout most of the period. Solidwood prices fall from an initial level of about $55 to $42 after 30 years terminating at about $44 in 2045. Pulpwood prices fall from $38 in year 0 to about $18 in year 50.

Scenario 2. Moderately High Demand (based on FAO forecasts)

This scenario examines the implications of a scenario which is loosely based on recent FAO forecasts. The FAO forecasts the demand of total industrial wood to increase by 1.8 percent annually and pulpwood to increase at a rate of 2.5 percent annually to the year 2010. In this scenario the growth period is extended to 2020, with the growth of demand declining linearly to zero in the year 2045, or after 50 years.

The results are presented in Figures 3.1 to Figures 3.5. Figures 3.1 and 3.2 present the model's projections of world and region pulpwood production for the high demand scenario. In this case 1995 pulpwood production is at 600 million cubic meters,[29] rising to 1.375 million cubic meters by 2045.

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[27] In order to get the system to solve for output from the responsive regions, the production of the non-responsive region was decreased by 0.5 percent annually the decline declining linearly to zero percent in year 50. In the absence of this adjustment, most of the production was generated by the non-responsive region.

[28] In a standard nature resource model, e.g., fisheries, a reduction in demand would result in a draw down of the initial stock to the new lower equilibrium level. In the TSM, however, demand is decreasing through time, but supply is also decreasing as several (11) of the land classes fall out of production due to the low prices. Thus, the stock has been adjust downward, largely through the reduction in price, which makes much of the original timber stock financially submarginal.

[29] It will be noted that the various scenarios give different initial 1995 production figures. This reflects the different expectations regarding future prices. If future prices are high, this expectation is reflected in lower current harvests levels as some of the harvest is postponed into the future when prices will be higher.

Although the pulpwood output total in 2045 is only about 50 million cubic meters above that of the base case,[10] overall industrial wood production approaches 2.6 billion cubic meters in 2045, about 300 million cubic meters above the output level of the base case. Figure 3.5 shows the behavior of relative prices. Both solidwood and pulpwood prices rise substantially throughout the 50 year period. Pulpwood prices almost double from the low $40 to the low $80. Solidwood prices more than double from about $60 to around $135.

Scenario 3. Very High Demand

This scenario posits a growth rate of demand twice that of the FAO scenario (scenario 2) above. In this case total industrial wood demand will initially growth at 3.6 percent and pulpwood demand at 5.0 percent to the year 2020. Thereafter both will decline linearly to zero at the end of fifty years in year 2045.

The results are presented in Figures 4.1 through 4.5. In this scenario initial annual production of pulpwood is very low, about 320 million cubic meters. This is because in our rational expectations model initial harvests are postponed in anticipation of higher future prices. Pulpwood production increases to a very high 2.2 billion cubic meters in year 50. Total world industrial wood production, figure 4.3, increases from an initial 1.1 to 2.9 billion cubic meters in year 50. The rate of growth of pulpwood demand outdistances that of solidwood so that total solidwood production (figure 4.4) declines from 780 to about 750 million cubic meters over the period. Finally, prices of both solidwood and pulpwood rise substantially throughout the 50 years period. Solidwood price rises from about $80 to around $310, while pulpwood prices rise from $60 to $310.

Regionally, most of the responsive regions postpone both pulpwood and solidwood production in the first two decades thereby allowing outputs to increase substantially in decades three and four. In decade five output growth flattens. Again, the major pulpwood producers are the US South and the Emerging region. However, all of the regions, including Eastern Canada and the Nordic region increase output substantially over the 50 year period. Western Canada, the Asia-Pacific and the Pacific Northwest show the lowest pulpwood production throughout the period but still expand production during the later years when prices are high.

Scenario 4. Integrated Supply Constraints with Decreasing Demand

The objective of this scenario is to examine the effects on output and prices of a highly constrained wood production system. In this scenario the following constraints will apply.

- 10 percent set-aside from all lands
- harvest costs increase by 20 percent
- no new plantation establishment
- no yield increases
- Eastern Canada has a harvest ceiling of 120 million cubic meters.
- US South has a harvest ceiling of 300 million cubic meters.

Scenario 4 applies the decreasing demand conditions of scenario 1 to the constrained supply side. The combined effects of the series of supply side constraints together with the

[10] The modest increase in pulpwood output reflects the relatively smaller increase in pulpwood demand compared to the more rapid rate of overall demand. This is reflected in the decline of the pulpwood price relative to solidwood over the period.

decreasing demand conditions severely limit wood production levels and also depress wood prices.

The demand growth in this scenario is the same as Scenario 1, Decreasing Demand. In this scenario total industrial wood demand is posited to decline at an rate of 1.0 percent in the initial year, linearly moving to zero percent demand change in year 50. Pulpwood demand is posited to decline at a rate of 1.5 percent in the initial years linearly moving to a zero percent demand change in year 50.

The results of this scenario, titled on the figures as the "Set Aside" scenario, appear in figure 5.1 through figure 5.5. World pulpwood production begins initially at about 570 million cubic meters falling over the 50 year period to 420 million cubic meters. Regionally, production is essentially stable or falling slightly for each region. The exception is the Emerging region in which the absence of new plantations and very modest investments in regeneration leads to sharper decline in pulpwood production. Overall world industrial wood production (figure 5.3) also declines, falling from 1.6 billion in year 1 to about 1.25 in year 50. As with pulpwood, world solidwood experiences a substantial decline (figure 5.4). Figure 5.5 presents the price projections and shows solidwood prices remaining essentially constant over the period at $58, while pulpwood prices declined from $45 to about $35.

Scenario 5. Integrated Supply Constraints with Base Case Demand

The objective of this scenario is to examine the effects on output and prices of the same highly constrained wood production system as examined in Scenario 4 combined with the demand growth posited in the base case.

Scenario 5 applies the modestly expansive demand conditions of the base case to a highly constrained supply side. The combined effects of the series of supply side constraints together with the modestly expansive demand conditions are to severely limit wood production levels even as demand continues to be expansive.

Figure 6.1 shows total global pulpwood expanding from 480 million cubic meters to 1.0 billion cubic meters at the end of 50 years. under these conditions the Emerging region and the US South are the major pulpwood supplying regions (Figure 6.2). Output continues to be maintained from the Emerging region due to the high prices which promote regeneration investment. In addition, the relatively high prices generate pulpwood output from all the regions except Asia-Pacific in the latter years.

World industrial wood (figure 6.3 increases from 1.3 billion in year 0 to 1.65 billion in year 50, while solidwood (figure 6.4) decreases modestly. Under the constrained supply conditions prices for pulpwood rises from $60 to $80 over the period and for solidwood from $77 to $87 (figure 6.5).

Scenario 6. Very High Demand with High Plantation Establishment

This scenario combines the very high demand conditions of Scenario 3, with a very high level of plantation establishment. A trial and error method was used to determine the level of plantation establishment required to stabilize pulpwood prices. It was conclude from this analysis that an annual plantation establishment level in the order of 6.5 million hectares would be needed for the first 25 years, thereafter declining linearly to zero by year 50.

The results are reported in figures 7.1 through 7.5. Total world pulpwood production increases dramatically from about 700 million cubic meters in year 0 to 4.3 billion in year

50. The dominant source of increase is the Emerging region, which swamps the other regions. The US South, Eastern Canada and the Nordic regions continue to be producers, with small volumes coming from some of the other regions. Total world industrial wood (figure 7.3) rises to almost 6.9 billion cubic meters and solidwood (figure 7.4) to about 2.5 billion in year 50 under this scenario.

Finally, pulpwood prices are relatively stable rising from $40 in the initial year to $53 in year 50, with the maximum price approaching $70 about year 40 (figure 7.5). Solidwood prices rise from $60 in the initial year to about $80 in year 50. Although not strictly constant, the prices are relatively stable, especially in light of the dramatic increases in production and consumption.

Conclusions

The above scenarios show that very large differences in output and prices are associated with large differences in the rate of demand growth and with large differences in the potential to produce and expand available supply. A summary of some aspects of the base case and various scenarios appears in Table 2.

Table 2. Summary of production and prices for the Base Case and Scenarios

	Production in year 50 (Million m3)		Prices in years 1 and 50 (US$ per m3)	
Scenario	Pulpwood	Total Industrial Wood	Pulpwood	Solidwood
Base Case	1,325	2,300	31 - 40	52 - 57
Decreasing Demand	570	930	38 - 18	55 - 44
Moderately High Demand	1,375	2,600	40 - 80	60 - 135
Very High Demand	2,200	2,900	60 - 310	80 - 310
Supply Constraints w/ Decreasing Demand	430	1,250	45 - 35	58 - 58
Supply Constraints w/ Base Case Demand	1,000	1,650	60 - 80	77 - 87
Very High Demand w/ High Plantation Estab.	4,300	6,900	40 - 53	60 - 80

The output of the base case suggest that modest increases in demand can probably be handled reasonably well by the existing market system of supply without requiring dramatically higher prices. However, as Scenario 5, the Integrated Supply Constraint scenario suggests, large constraints on supply potential are likely to result in both lower outputs and substantially higher prices. Finally, Scenario 6 suggests that really dramatic increases in demand can, in principle, be handled by increasing the rate of plantation establishment. However, for the demand increases posited, an order of magnitude increase, to over 6 million ha annually, in plantation establishment would be required. Nevertheless, such an increase is probably possible in principle as large areas of land seem to be available in parts of the tropics (see Grainger 1988) and also in semi-tropical regions of South America and Asia.

Some of the outputs of this model may be difficult to accept in the "real world" context. These can better be viewed as lessons or implications as to the type of logical consistencies required.

1. One of the important lessons of the analysis relates to the relation between pulpwood demand and total industrial wood demand. If pulpwood demand grows at a significantly higher rate for a long period of time, solidwood consumption will stagnate and ultimately decline. Upon reflection, this is intuitively obvious but easy to overlook. The model reinforces the logical implications of the underlying trends.

2. If supply is constrained in some regions the result will be increased harvest pressures on other regions and harvest levels beyond what might normally be considered realistic.

3. In scenarios that involve substantially higher future demand the initial harvest is likely to be substantially reduced in order to provide more harvest in the future, when prices are high. In essence, harvests are shifted from periods of low demand to periods of high demand in order to maximize returns. This adjustment also tends to flatten the price trend lines.

Discussion by Region

US South

The US South is a major industrial wood producer, including importantly pulpwood. The South produces both conifer and hardwood pulpwood from plantations, managed natural regenerated forests and un-managed naturally regenerated forests. The capacity for increased future harvests comes from more intensive management and increased harvests from the surplus of hardwood timber, the utilization of lower quality and less accessible forests, as well as the possibility of increased forest from croplands that revert or are converted to forestry.

The base case projections show that the US South is the major pulpwood producer today, at the beginning of the 50 year period, and the projections indicate that the US South will continue to be an important and an expanding producer of pulpwood. This projected performance is consistent with the large areas of available land, good tree growing conditions including accessible terrain, an accessible and well developed transport infrastructure, and the large ongoing investments in tree growing that are being made and have been made over the past three decades. The South is also in a strong position to shift production between pulpwood and solidwood since by extending the rotations 5-10 years, many pulpwood stands can be converted into solidwood. Similarly, by reducing rotations lengths, more wood can be shifted into pulpwood.

The scenario analysis indicates that the South will be a major producer of pulpwood under almost all of the proposed scenarios. However, a large increase in Emerging region plantations would have a damping impact on the expansion of the South's production, as shown in Scenario 6. The lower prices associated with high plantation production would discourage investment and production in the South.

US Pacific Northwest

The original projections of the TSM reported in the Sedjo and Lyon book of 1990 anticipated a major timber production role for the PNW. However, policy changes that involved large reductions in harvests from public lands were captured in the modified reassessment undertaken in 1994 (Sedjo et al.) In that assessment adjustments were made in harvestable inventories in the PNW to accounted for the public lands set-asides. The PSM incorporate those 1994 changes in the TSM. Thus, pulpwood production in the PNW is expected to be relatively modest in the future due to the US policy which has severely restricted timber harvests from public lands, especially in the PNW. The results of the scenario analysis are generally consistent with this outlook, except for the extreme scenarios.

Pulpwood production in the PNW has been largely the result of residues that are created in the sawmilling process and reduced harvests, other things equal, imply reduced residues for pulpwood production. In the future these residues are expected to be supplemented by thinnings. However, the scenario analysis indicates that pulpwood production falls when pulpwood prices are low relative to solidwood as in the high plantation and low demand scenarios. This outcome reflects, in part, the region's ability to modify its sawmilling technique to produce more solidwood and less pulpwood.

Canada, west

Most of the pulpwood in western Canada comes from harvests of original forests. Along the coast the trees are relatively large. However, in the interior regions the trees are smaller and, in the northern areas, are useful largely only for pulpwood.

In the future, pulpwood production in the Canadian west is projected to be modest, in part due to the proposed reduction in harvests in British Columbia. As with the policy changes in the US PNW, the changes in BC policy were incorporated into the 1994 TSM model as a reduction in accessible timber. These modifications are also used in the development of the PSM. However, there is likely to be some offsetting of the reduction in BC by increased harvests in other western provinces, such as Alberta.

In our scenarios the role of western Canada as a pulpwood producer on a world scale is always modest. Under decreasing demand conditions pulpwood output may fall to very small, almost negligible, levels.

Canada, east

As with western Canada, most pulpwood in eastern Canada comes from harvests of original old growth forests. The resource base involves vast tracts of native species. The trees are relatively small, and a large portion are mainly suitable for pulpwood.

Pulpwood production in the Canadian east is significant in the base case. Under most situations of expanding demand and higher prices the role of Eastern Canada as a pulp producer continues to be substantial. However, pulpwood production in eastern Canada is quite sensitive to pulpwood prices. Under conditions of falling prices, as in the decreasing demand scenario, the role of the Canadian east as a pulpwood producer can decline greatly.

Emerging Region

The emerging region consists of a host of countries that have not traditionally been major industrial wood producers. The emerging regions countries tend to be located in the tropics or semi-tropics, utilize non-indigenous species in a plantation mode, and have short-rotation fast tree-growing conditions. These countries include Australia, New Zealand, Chile, Brazil, Spain, Portugal, South Africa, Indonesia and others. The focus on short rotations and fast growth suggests that the output of this region will be predominantly in pulpwood rather than solidwood.

The emerging region is a substantial producer of pulpwood today, with much of the growth coming within the last 15 years (Sedjo 1995). Over that period this region has increased its exports of pulp from seven to 20 percent of the world export market. During this time the total market expanded by 43 percent.

The base case projects substantial production increases over the next five decades, essentially doubling its total pulpwood production. However, the scenarios show that this expansion could be substantially greater than projected in our base case if plantation establishment proceeded at a more rapid rate than assumed. In Scenario 6 the rate of plantation establishment was increased by a factor of 10 over the base case in the face of a very rapid growth in demand for industrial wood and pulpwood. In this scenario the output of the emerging region dominates world production.

Nordic Region

The Nordic region consists of Finland, Sweden and Norway, with the first two being large producers of pulpwood. The majority of the pulpwood harvested in the Nordic region is conifer from managed naturally regenerated stands, with lesser amounts from managed plantation forests (WRI). Finland and Sweden supplement their domestic production of pulpwood with significant additions from imports. The major foreign suppliers are countries of the former Soviet Union.

The Nordic region is a significant producer of pulpwood and our projections indicated that this will continue to be the case under all of the scenarios. Our projections indicate that Nordic production is relatively insensitive to the various scenario conditions, unless they are extreme. Under most scenarios Nordic pulpwood production is significant throughout the 50 year period under examination.

Asia-Pacific

The Asia-Pacific is only a very modest producer of pulpwood, currently less than 5 million cubic meters, and our projections see this situation continuing. The pulpwood source today consists of some plantation wood but mostly mixed tropical hardwood species. Our projections of the Asia-Pacific in the TSM and the PSM are based almost exclusively on harvests from the native forest and do not assume that large areas of new plantations will be established nor that there will be large increases in the use of mixed tropic timber pulpwood.

However, it should be noted that Indonesia does have very ambitious plans for the future creation of some 32 pulp mills by 2020. However, in our analysis the plantations that would be required to provide the pulpwood feed stock for these mills are incorporated in the Emerging region.

Rest of the World

The "rest of the world" consists of all pulpwood producers not covered by the seven responsive regions and includes most of Europe (except the Nordic countries), the former Soviet Union, China, Japan and other major producers. Substantial changes have occurred in some of these regions since the development of the TSM, especially in eastern Europe and the former Soviet Union. In the former Soviet Union, for example, industrial wood harvests are reported to have declined as much as 50 percent during the earlier part of the 1990s, although some recovery has subsequently taken place.

Overall, our base-case projections have the rest of the world pulpwood production being about 20 percent of the total in 1995 rising to about 33 percent in 2045. The rise in the share of the rest of the world is attributed to the resurgence of pulpwood production over the period by the former Soviet Union, especially Russia.[31]

Comparative Advantage

Comparative advantage is a situation in which a region's endowment of resources or factors of production give it a sufficient cost advantage in production so that it becomes a net exporter of the product. For pulpwood a comparative advantage would be reflected by the net export of pulpwood or the net export of wood pulp that is produced using domestic pulpwood. Commonly, a region with abundant pulpwood resources will also process the pulpwood into pulp and often into paper.

All of the seven responsive regions are known to be major producers and exporters of industrial wood, pulpwood or pulp. This fact is evidence that the regions have a comparative advantage in industrial wood production.

An examination of the base case reveals that six of these regions - the US South, the US Pacific Northwest, the Emerging region, eastern Canada, western Canada, and the Nordic region - all show substantial regional net exports of pulpwood, pulp and/or paper.

However, some of these regions show declining production and a fall share of pulpwood production in the face of decline prices. For example, both eastern and western Canada see a decline in their share of the world's pulpwood market in the face of decreasing pulpwood prices as in scenario 1. Also, the Asia-Pacific's modest role and to some extent that of the Pacific Northwest decline in the face of lower prices. This suggests that they are likely to experience an erosion of their comparative advantage should pulpwood real prices experience long-term declines.

By contrast, in an environmental of declining prices the market share of the Emerging region appears to increase. Similarly, both the US South and the Nordic region maintain their share indicating an ability to continue to maintain a comparative advantage.

International Trade Implications

This section examines the international trade implications of some of the projections of the PSM. Our PSM has treated world demand as being one of three huge markets: the northeastern US and eastern Canada in North America; western and central Europe; and East Asia. As noted in the section on "comparative advantage," we expect that at least five of the seven responsive regions will continue to be the major exporters of pulpwood or regionally processed pulpwood and/or products.

Pulpwood products are traded internationally in three forms. These are raw wood, i.e., pulpwood logs and chips, pulp, and paper products including newsprint. Major international flows of raw pulpwood have been largely confined to flows into Japan from the PNW and western Canada in North America and from Australia. Recently there have been some flows of pulpwood from the Gulf Coast in the US South to Japan. In addition, there have been continuing pulpwood flows from the former Soviet Union to Finland.

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[31] If future pulpwood demand increases more rapidly than solidwood demand, we would expect that most if not all regions would increase their relative production of pulpwood.

The second form of international flows is in the form of market pulp from pulp mills, typically located near the source of pulpwood, to paper mills, located throughout the world. These trade flows are common of the emerging region where the pulp is exported to paper mills located near large markets. Traditionally pulp flows from the Nordic countries to Europe; from eastern and western Canada to Europe, East Asia and the US; and from the western US to Asia and eastern North America. In recent decades Japan has imported large volumes of pulp for domestic paper mills.

A third international flow is in the form of paper products. An example would be the increasing portion of paper product exports from the Nordic region to western and central European markets. An interregional example would be the paper products flow from pulp-paper mill complexes in the US South to various other regions of the US including the Northeast-North Central region.

In general, we expect to see most of the existing trade flows persist through the first half of our fifty year projection in the absence of the occurrence of radically altered conditions, such as those of scenarios one and six. In all likelihood, Europe will continue to import pulp and paper from the Nordic countries as well as from Canada and the emerging region. Increased pulpwood supplies might be expected from eastern Europe. At some future time it would be expected that Russia may become a major wood supplier, perhaps initially through pulpwood and later through pulp. However, it is currently impossible to place a time dimension on these activities

Historically, North America has been a large net exporter of pulpwood and pulp. However, with the conservation set-asides already in place that will restrain future production in the western US and western Canada, a greater share of North American production is likely to be required to meet domestic demand. Wood chip exports to East Asia from western Canada and the PNW are likely to decline. Furthermore, the role of the US South and eastern Canada in supplying north American pulp requirements is likely to increase to fill voids that result from decreased harvests in the west.

An important trade change that we anticipate is the increasing export of pulp from the Emerging region as it expands over the fifty year period. Much of the plantation activity has been in species that have potential for pulp and pulp mills are invariably part of the long-term planning associated with plantation establishment. Thus we would expect, and indeed are witnessing increased market pulp flows from countries in the Emerging region including Chile, South Africa, Brazil and others. The markets for this increased flow of pulp are likely to be Asian markets, as well as Europe and, to a lesser extent North America.

Additionally, we expect to see some decrease in the flows of pulpwood and chips from North America to East Asia and especially Japan. This anticipated decrease would be the result of two trends: First, North American pulpwood surpluses are likely to be reduced due to environmental constraints. Second, Japan is likely to shift from producing its own pulp to relying increasingly on imported pulp and paper.

Finally, it should be noted that the increased recycling of waste paper is a substitute for virgin wood fiber in the pulping process. Thus, increased use of recycled paper is a direct substitute for wood pulp in many uses. Thus, waste paper flows can be a substitute for pulpwood or chip flows.

References

Wood Resource International Ltd. 1995. "Fiber Source Analysis for the Global Pulp and Paper Industry," Consultant report to IIED, June.

Grainger, A. 1988. "Estimating Areas of Degraded Tropical Lands Requiring Replenishment of Forest Cover," *International Tree Crops Journal* Vol.5 pp. 1-2.

Sedjo, R.A. 1995. "The Potential of High-Yield Plantation Forestry for Meeting Timber Needs: Recent Performance and Future Performance," RFF discussion Paper 95-08, 30 p, December, Resources for the Future, Washington.

Sedjo R.A. and K.S. Lyon. 1990. *The Long-Term Adequacy of World Timber Supply*, Resources for the Future, Washington, DC.

Sedjo R.A., A.C. Wiseman, D.J. Brooks and K.S. Lyon. 1994. "Global Forest Products Trade: the consequences of domestic forest land-use policy." Discussion paper ENR 94-13, Resources for the Future, Washington, DC

Figure 1.1 World Pulpwood: Base Case

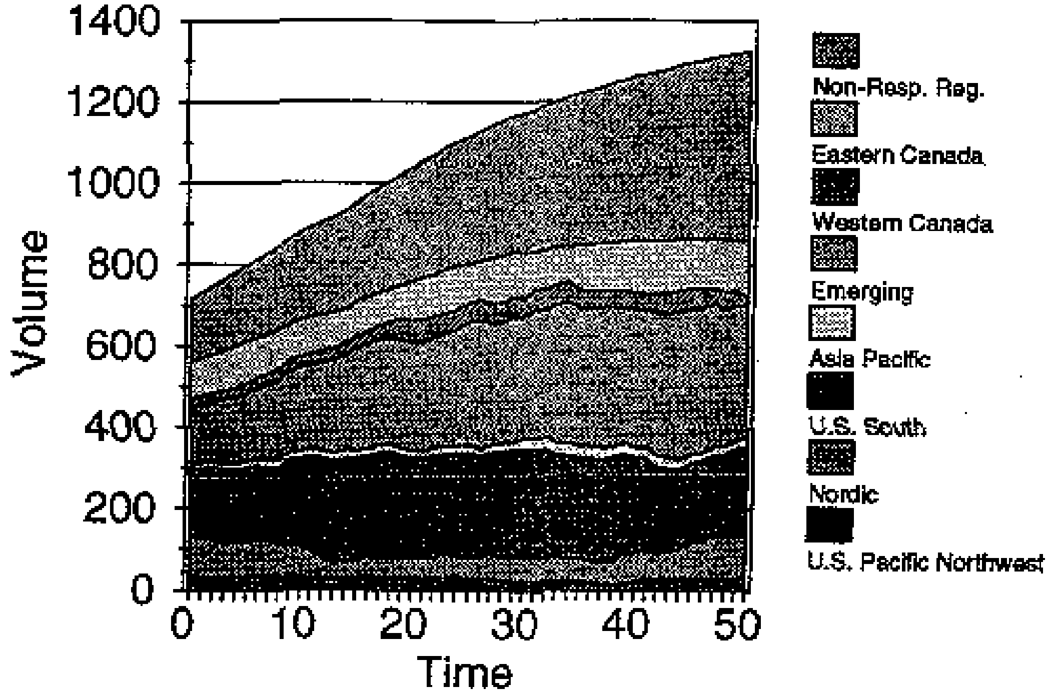
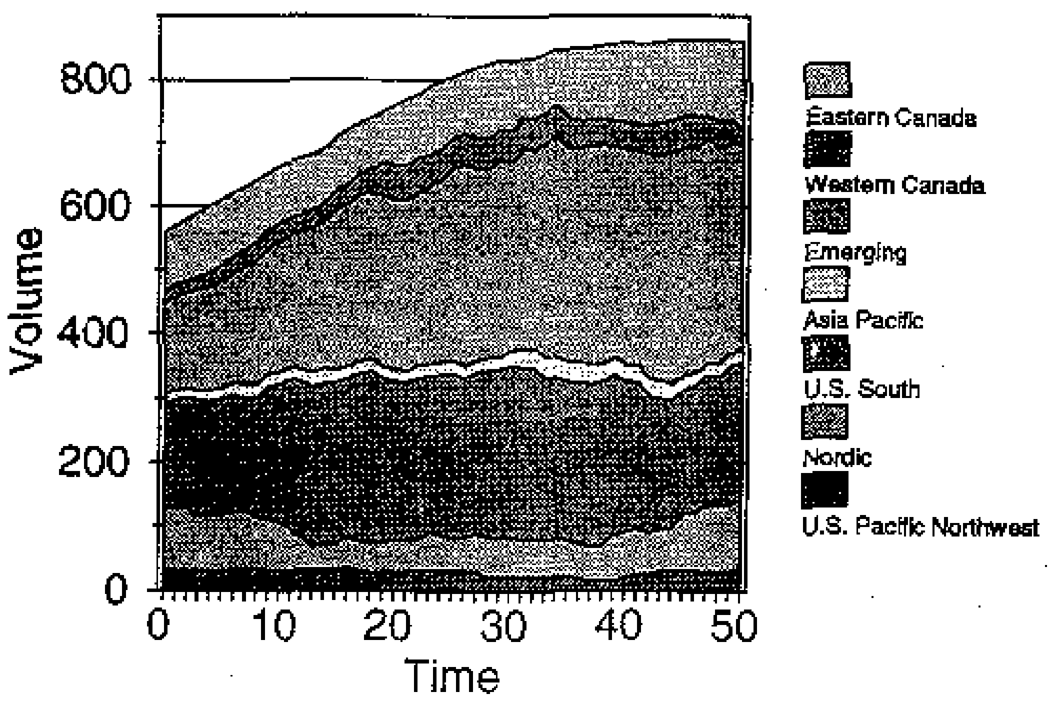


Figure 1.2 Pulpwood: Base Case



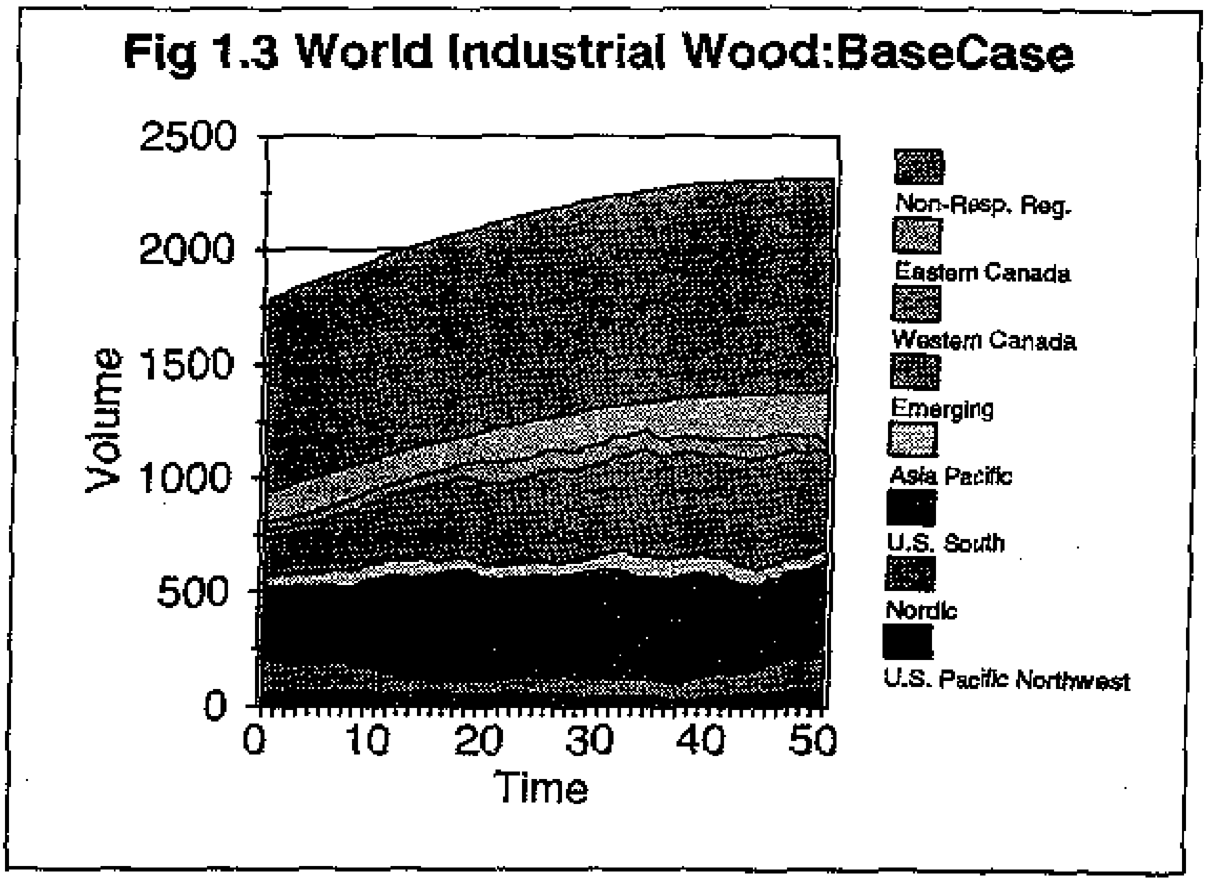


Fig 1.3 World Industrial Wood:BaseCase
Volume
2500
2000
1500
1000
500
0
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Figure 1.4 World Solidwood: Base Case

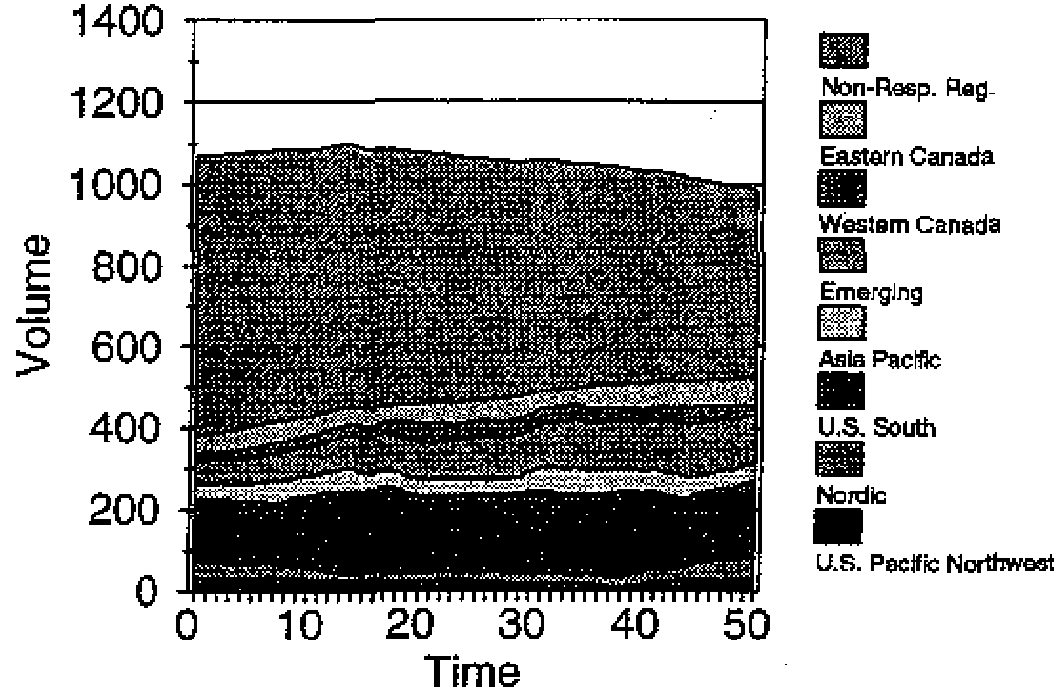


Figure 1.5 Prices: Base Case

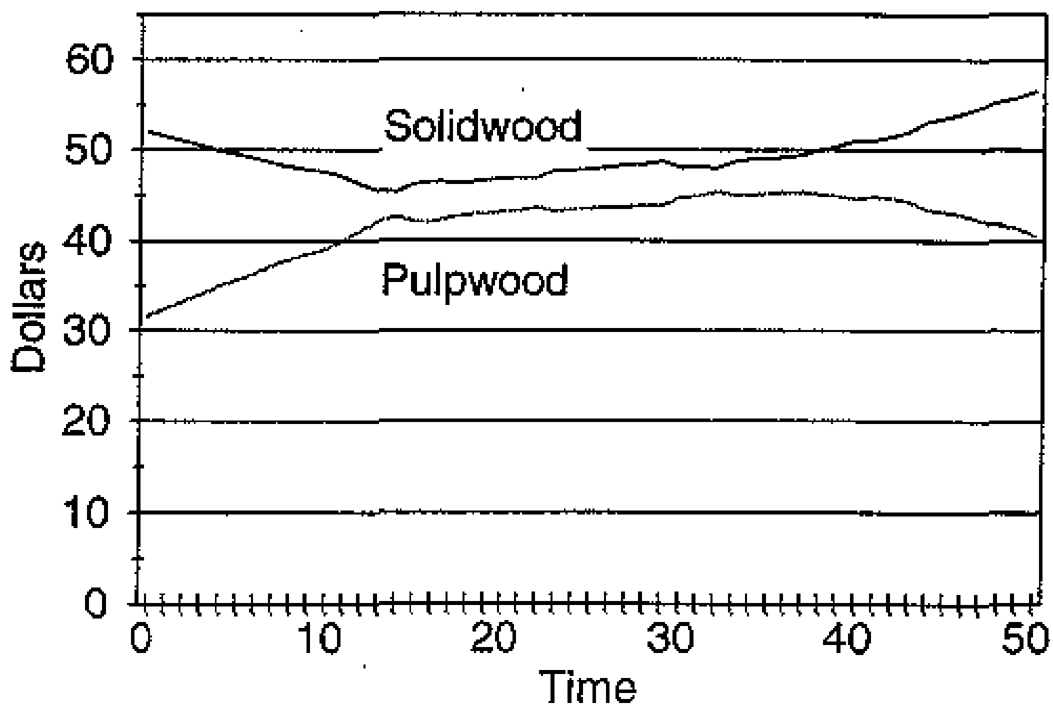


Fig 2.1 World Pulpwood:Decrs Demand

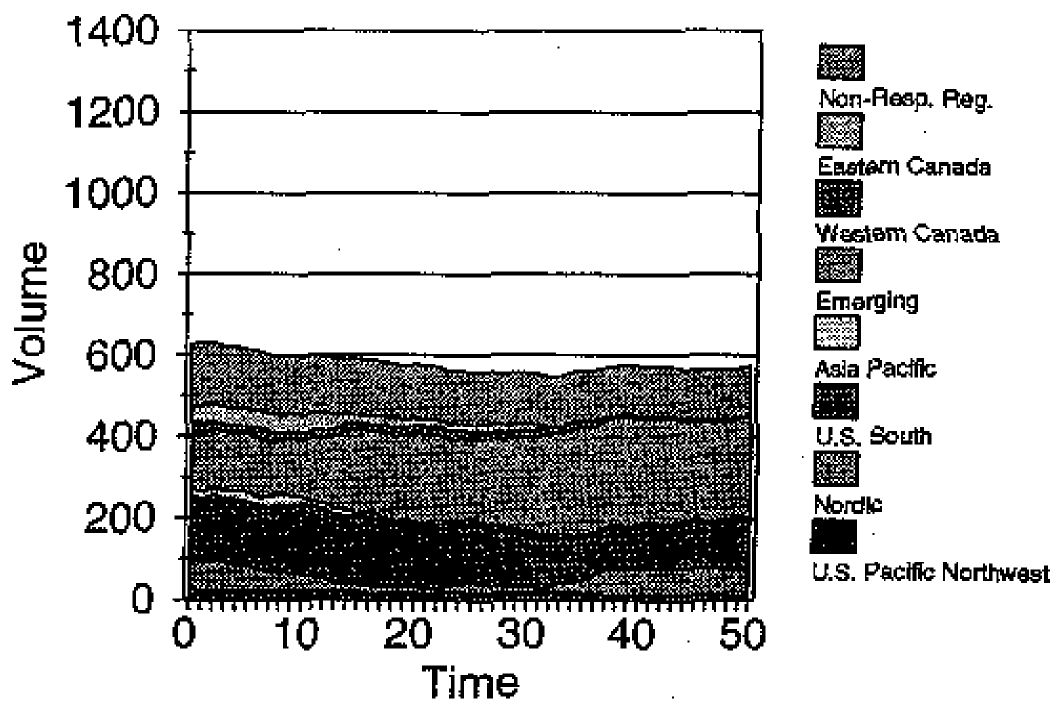


Fig 2.2 Pulpwood:Decreasing Demand

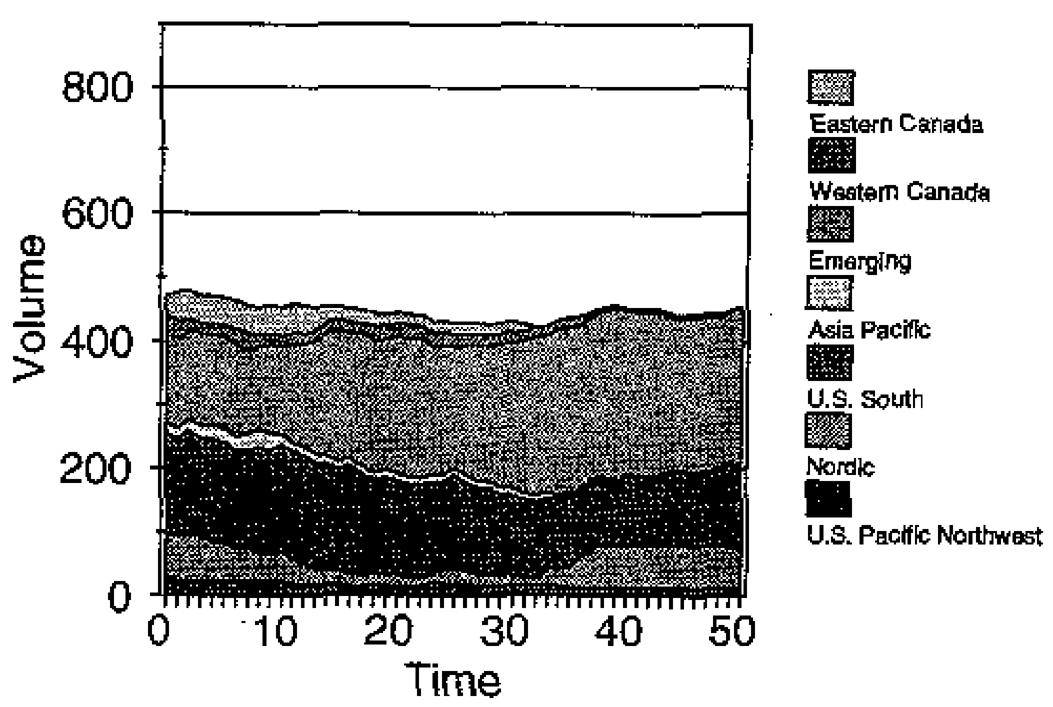


Fig 2.3 World Ind Wood:Dcrs Demand

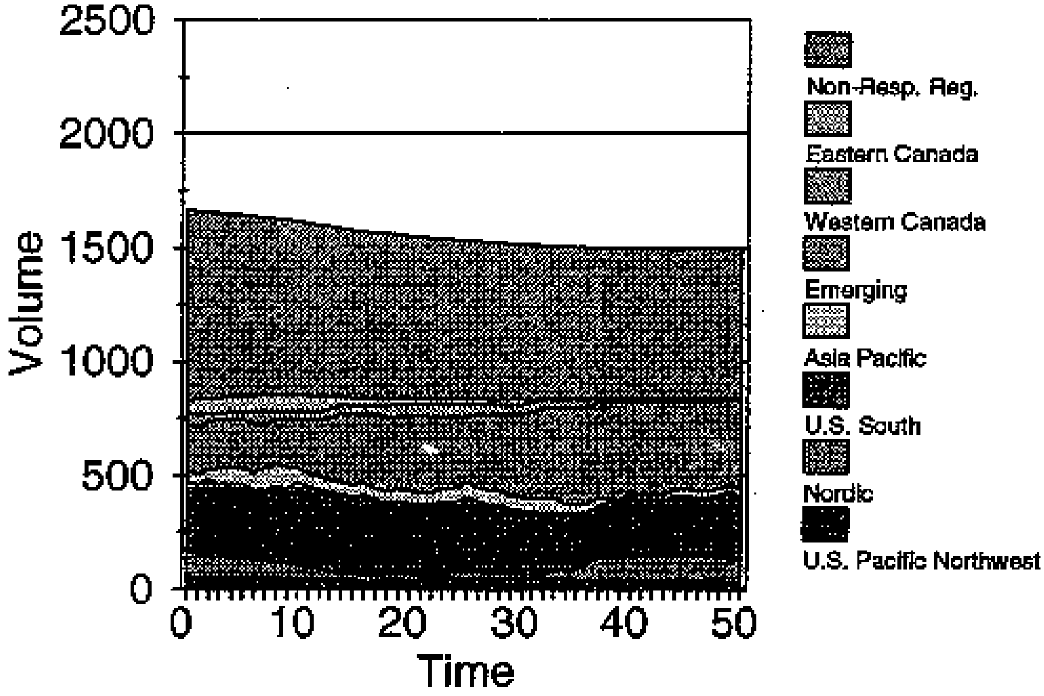


Fig 2.4 World Solidwood:Dcrs Demand

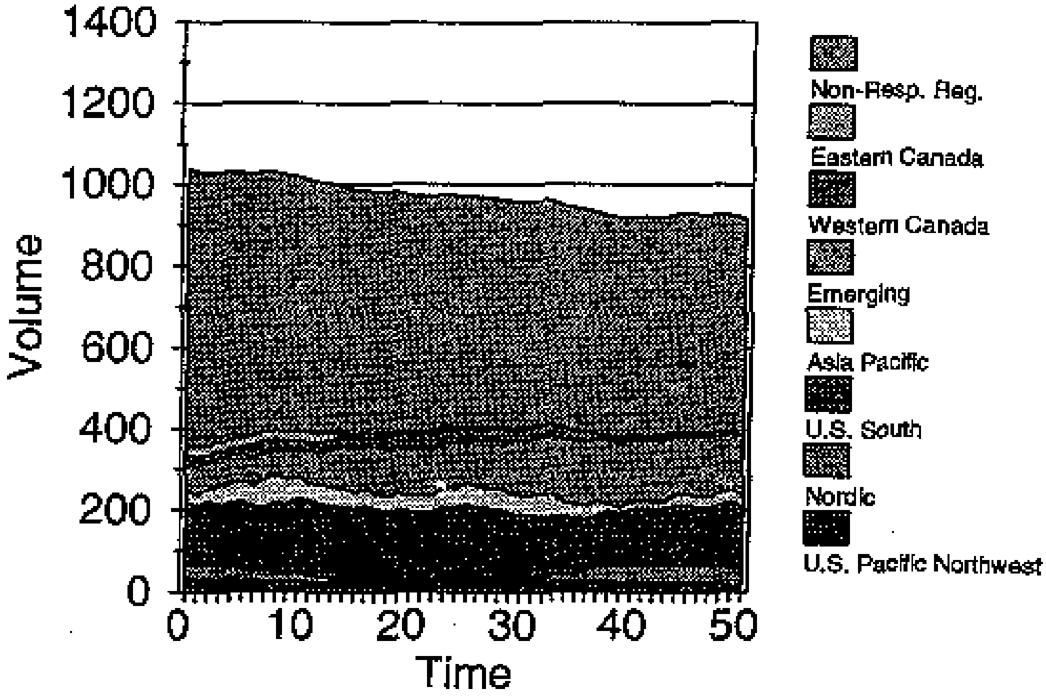


Fig 2.5 Prices:Decreasing Demand

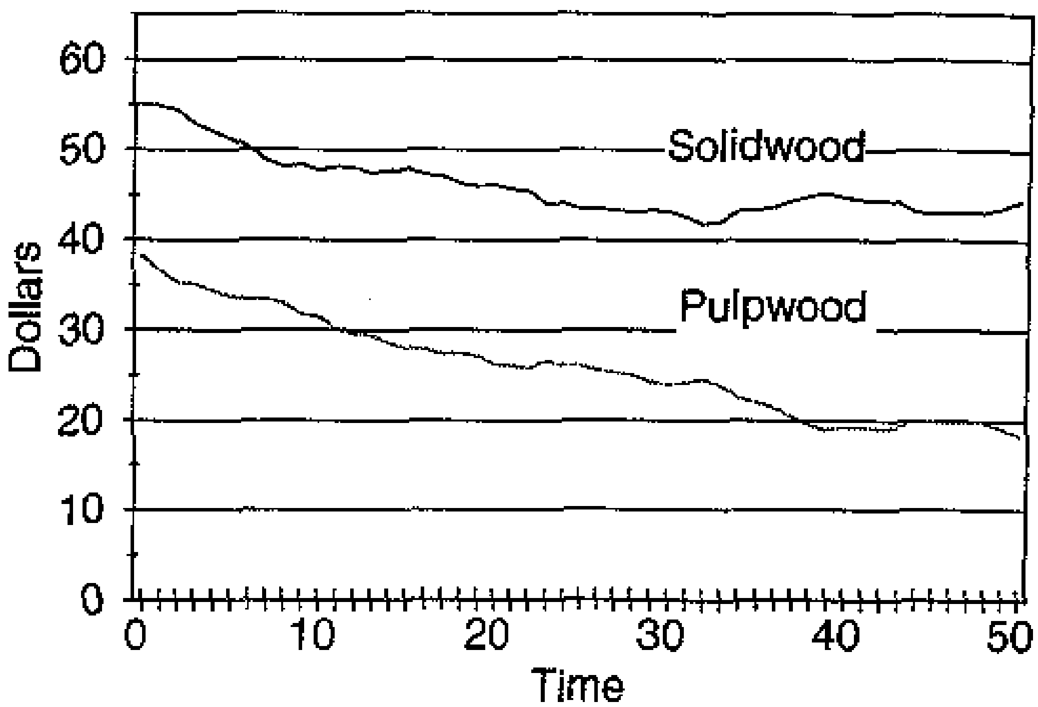


Fig 3.1 World Pulpwood: High Demand

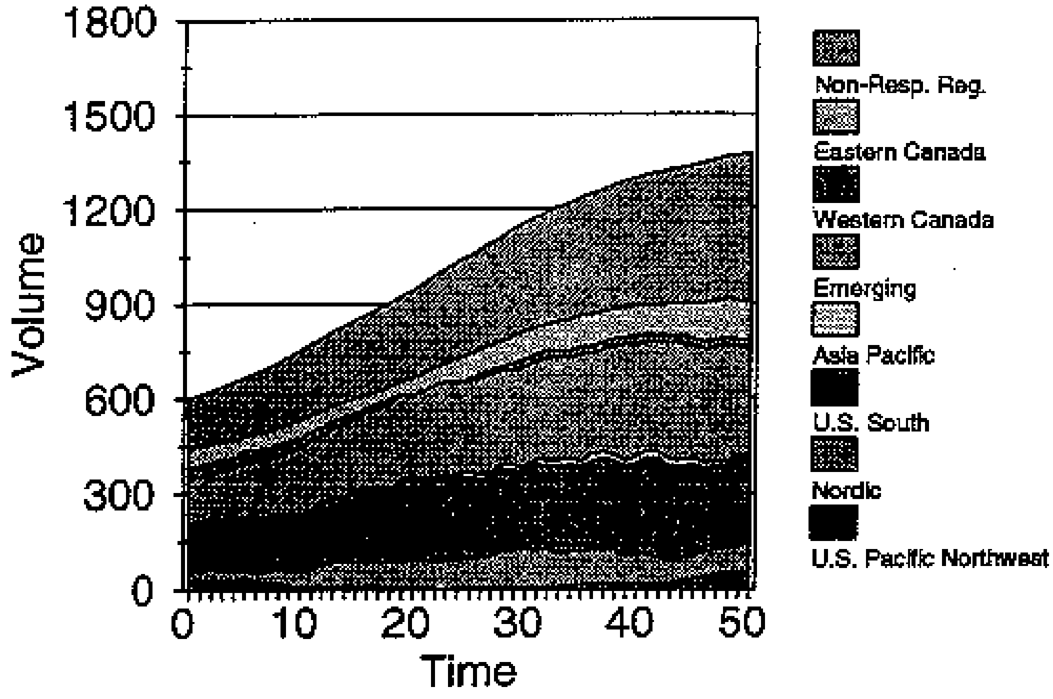
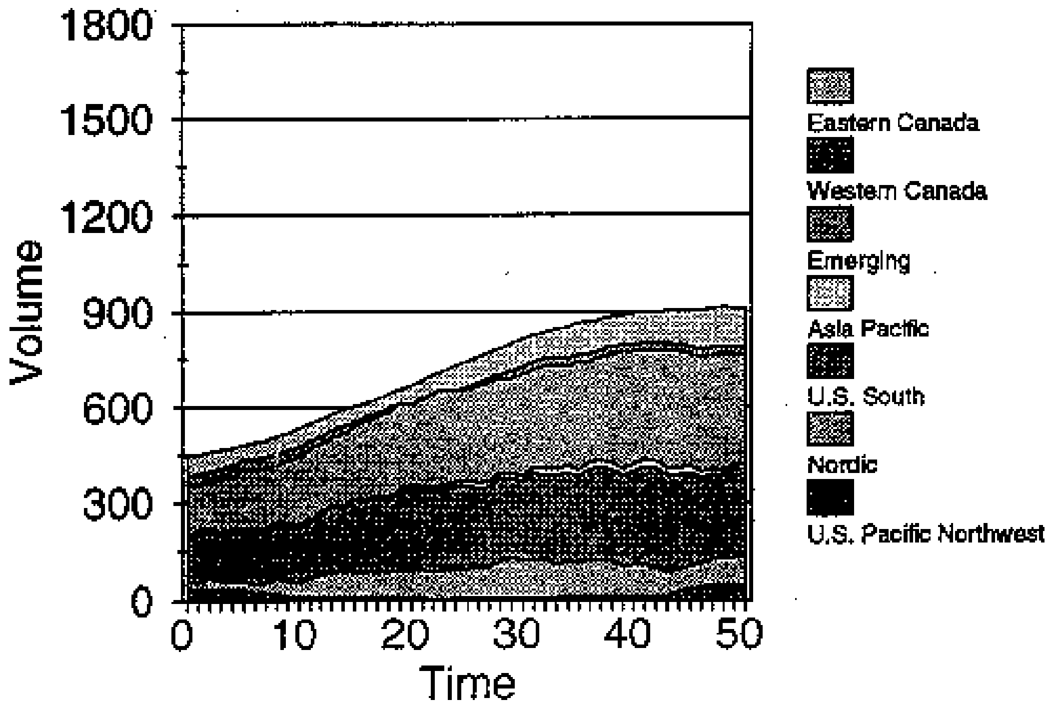


Fig 3.2 Pulpwood: High Demand



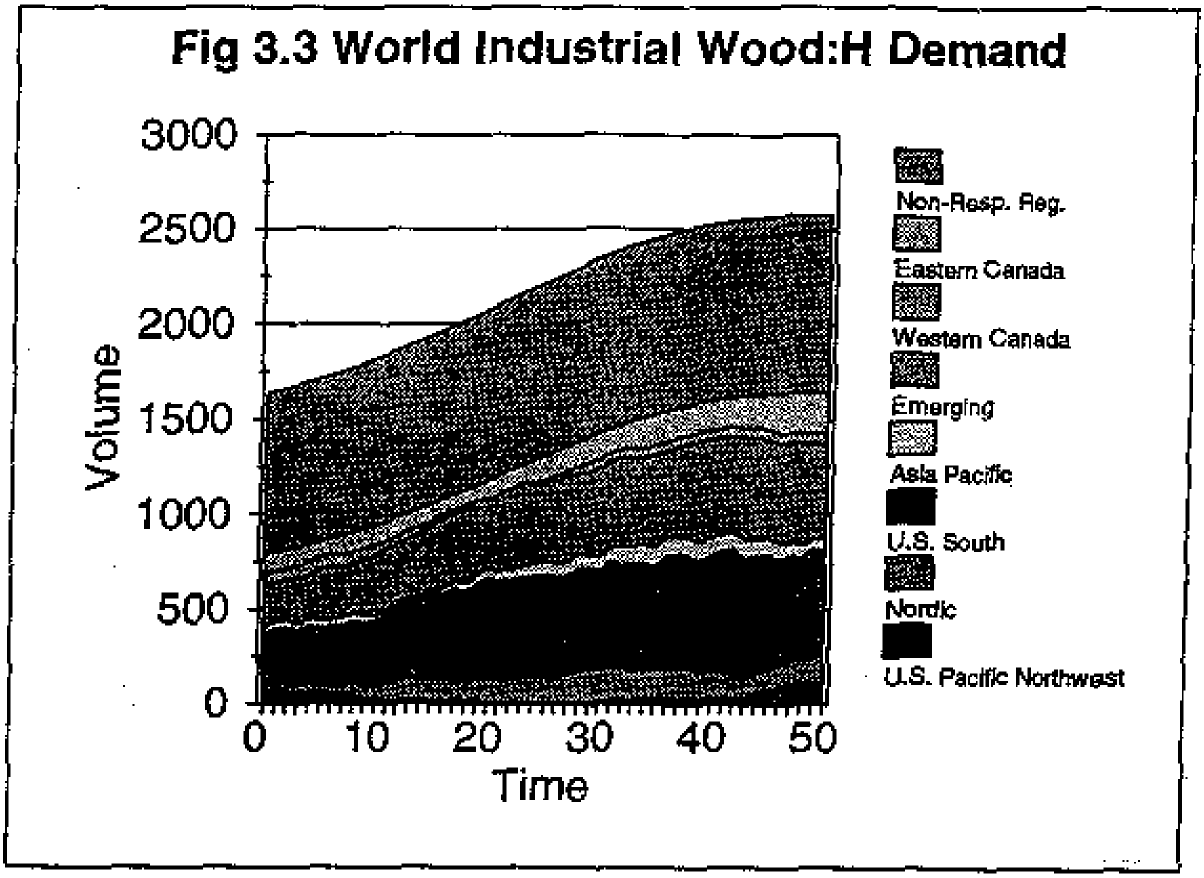


Fig 3.3 World Industrial Wood:H Demand
3000
2500
2000
1500
1000
500
0
Volume
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Fig 3.4 World Solidwood: High Demand

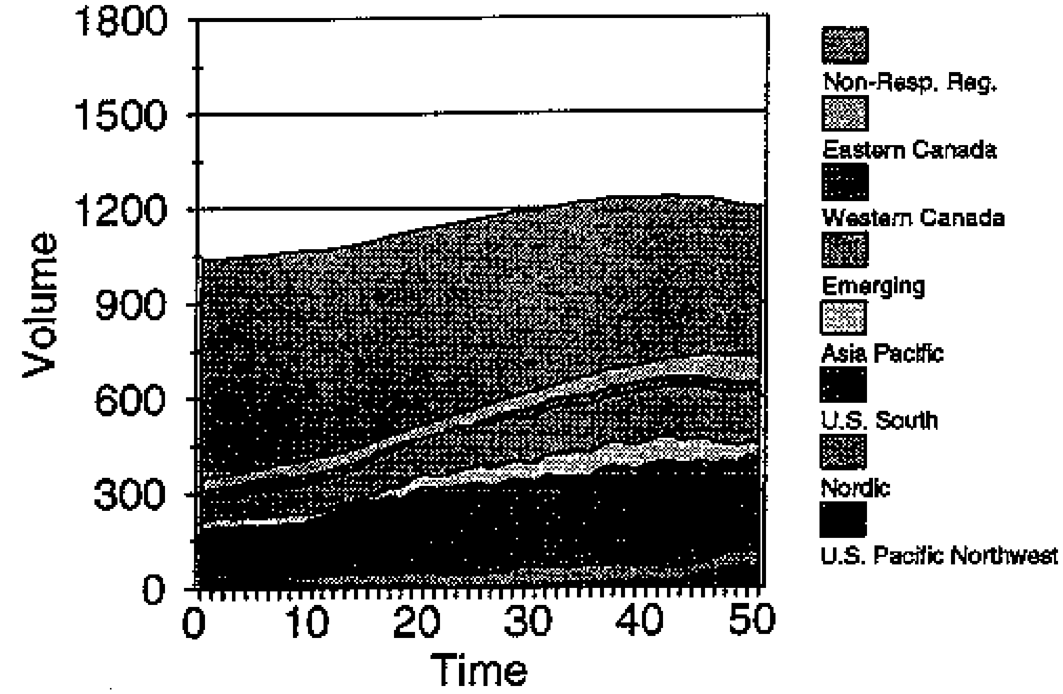


Fig 3.5 Prices: High Demand

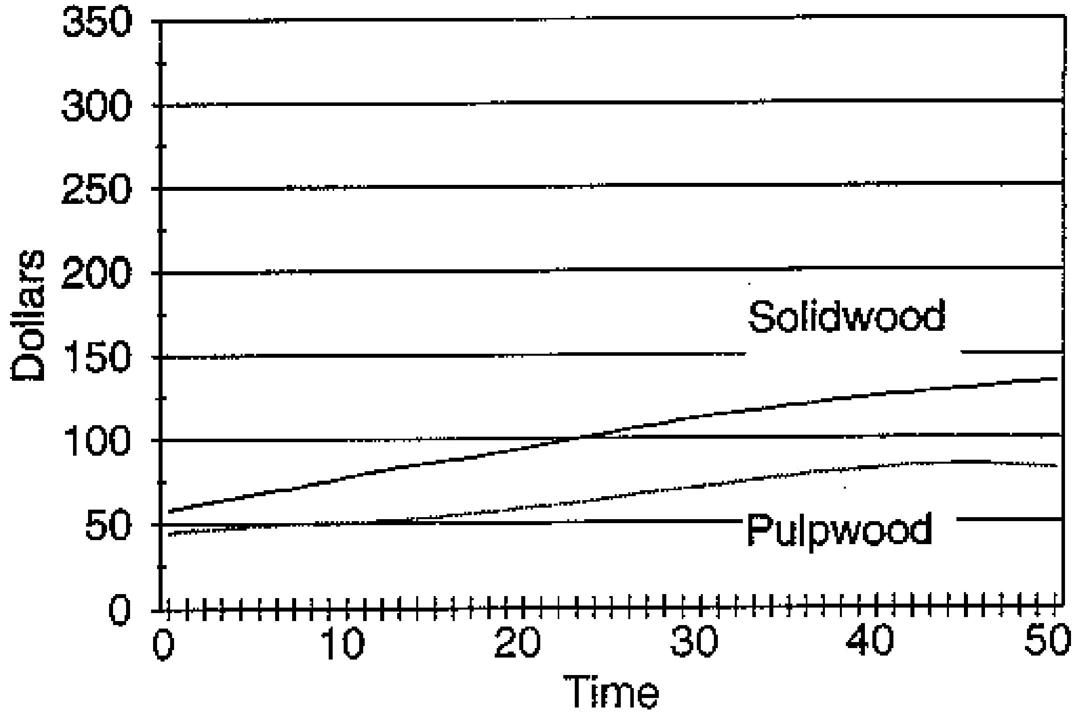


Fig 4.1 World Pulpwood:V High Demand

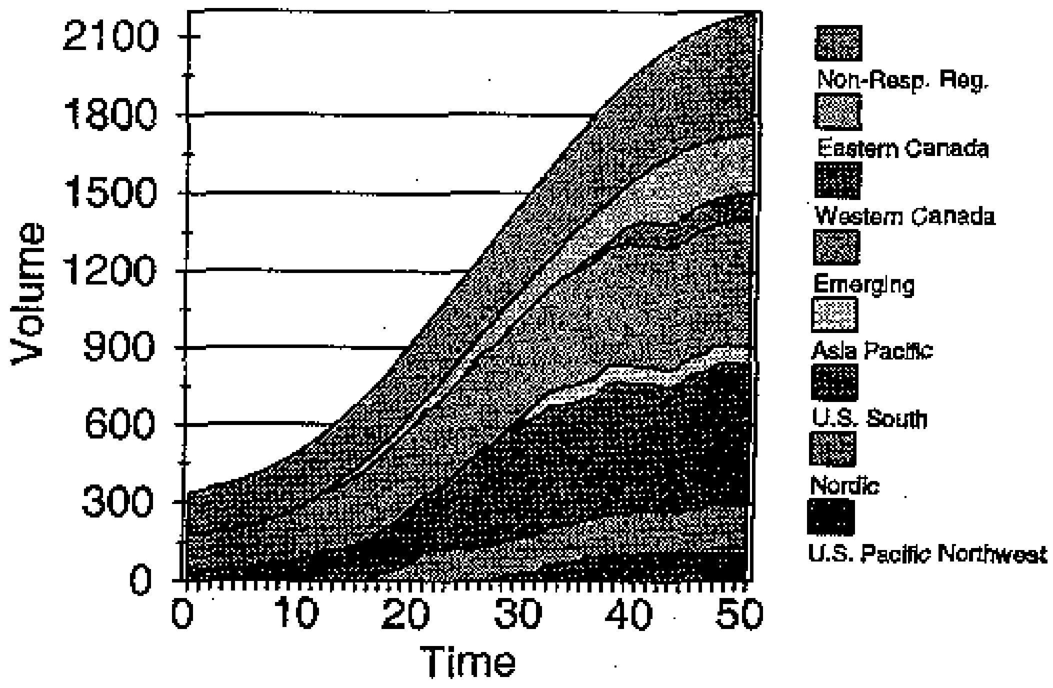
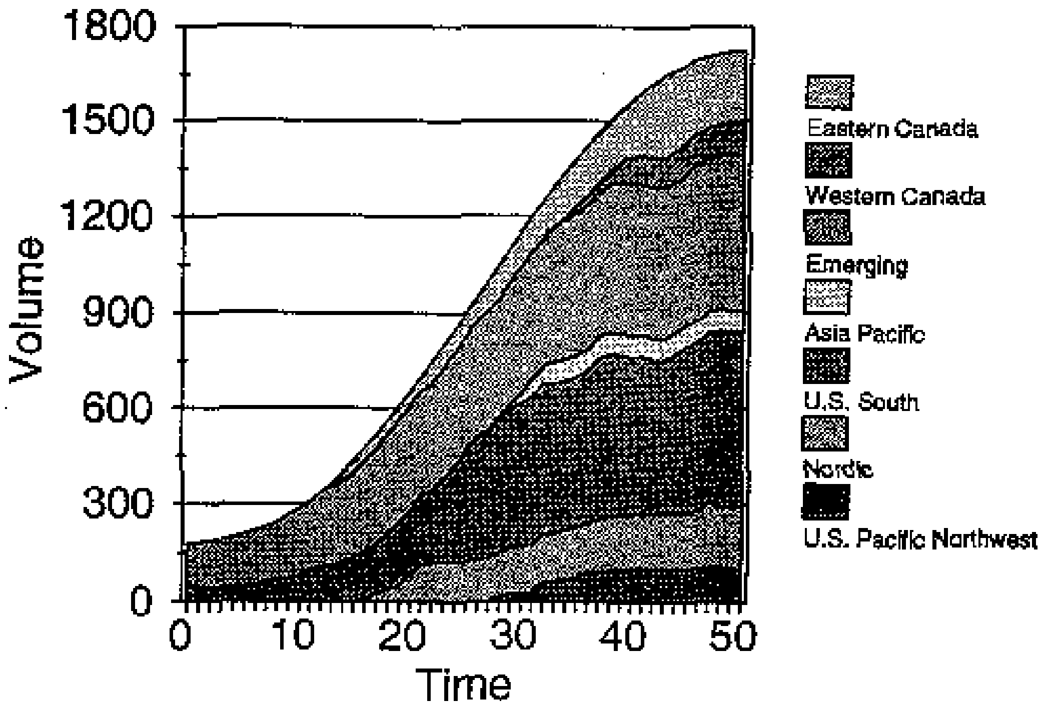


Fig 4.2 Pulpwood:Very High Demand



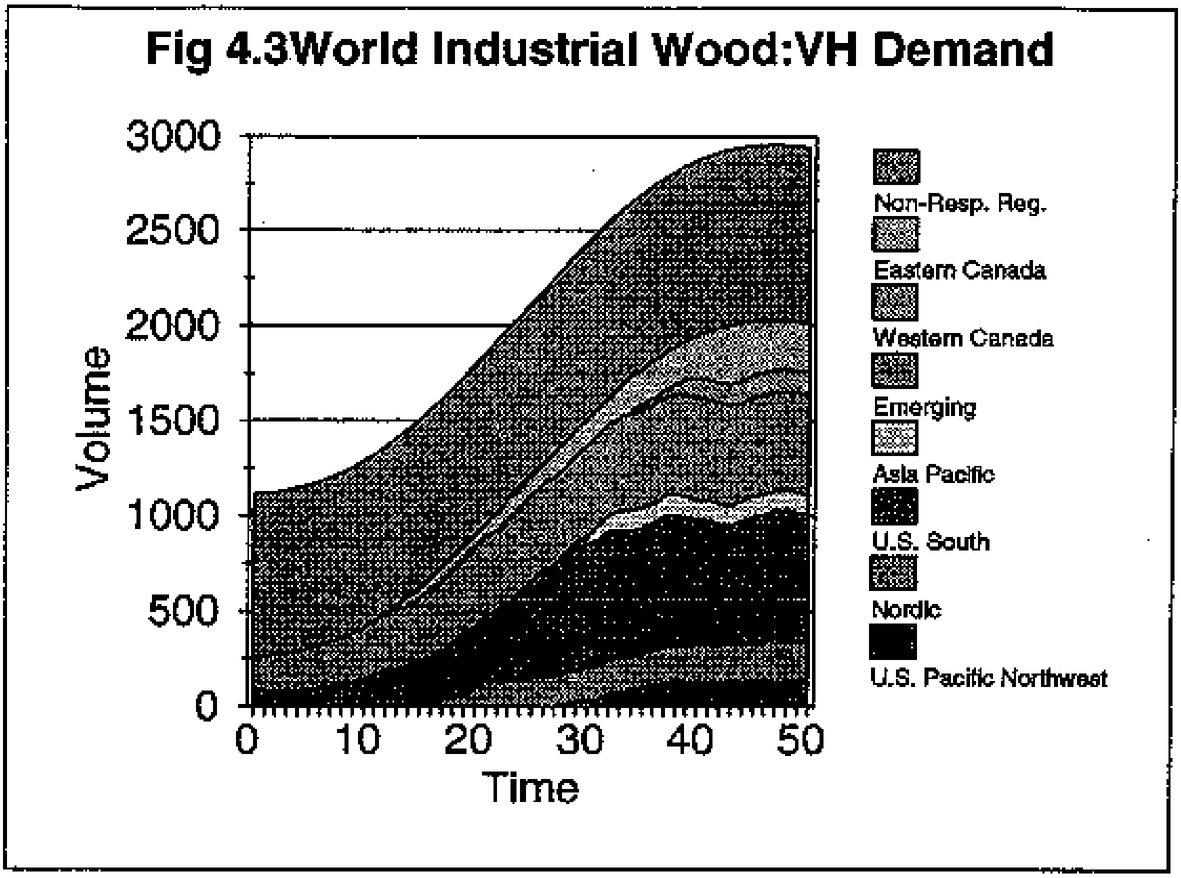


Fig 4.3World Industrial Wood:VH Demand
3000
2500
2000
1500
1000
500
0
Volume
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Fig 4.4 World Solidwood:V High Demand

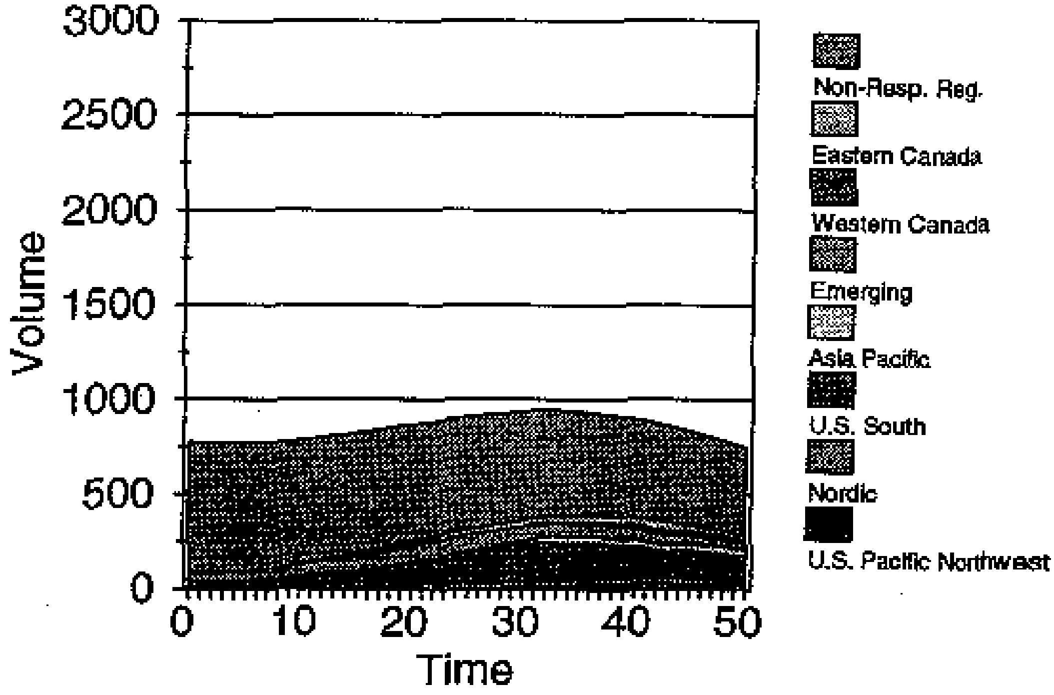


Fig 4.5 Prices:Very High Demand

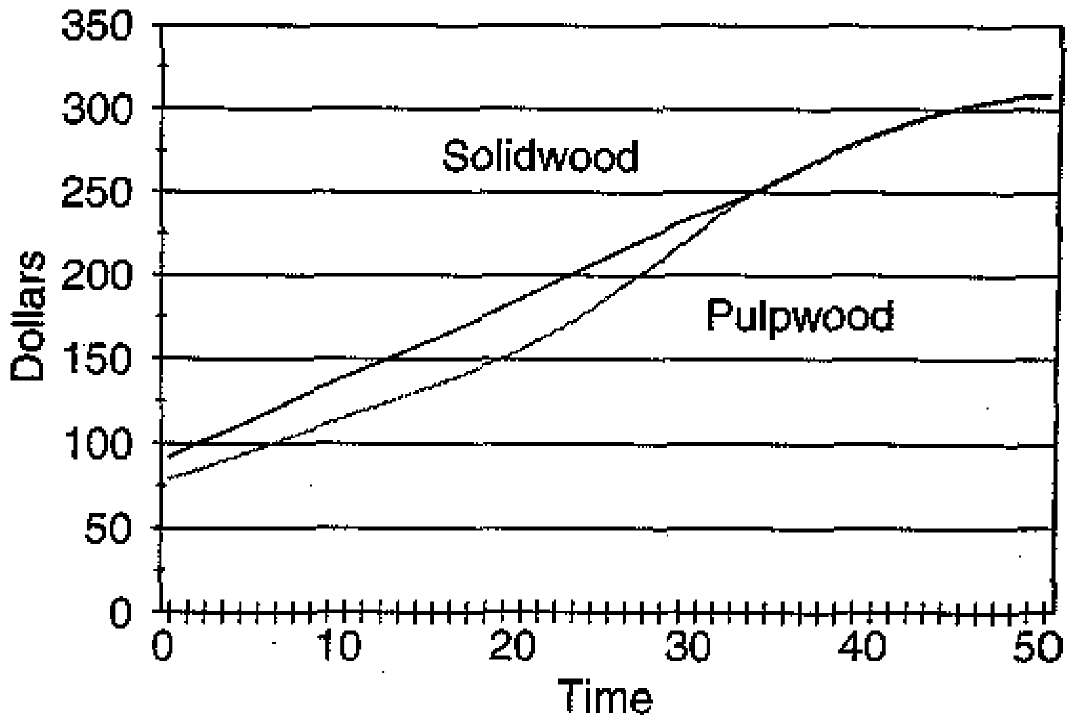


Fig 5.1 World Pulpwood: Integ Constr

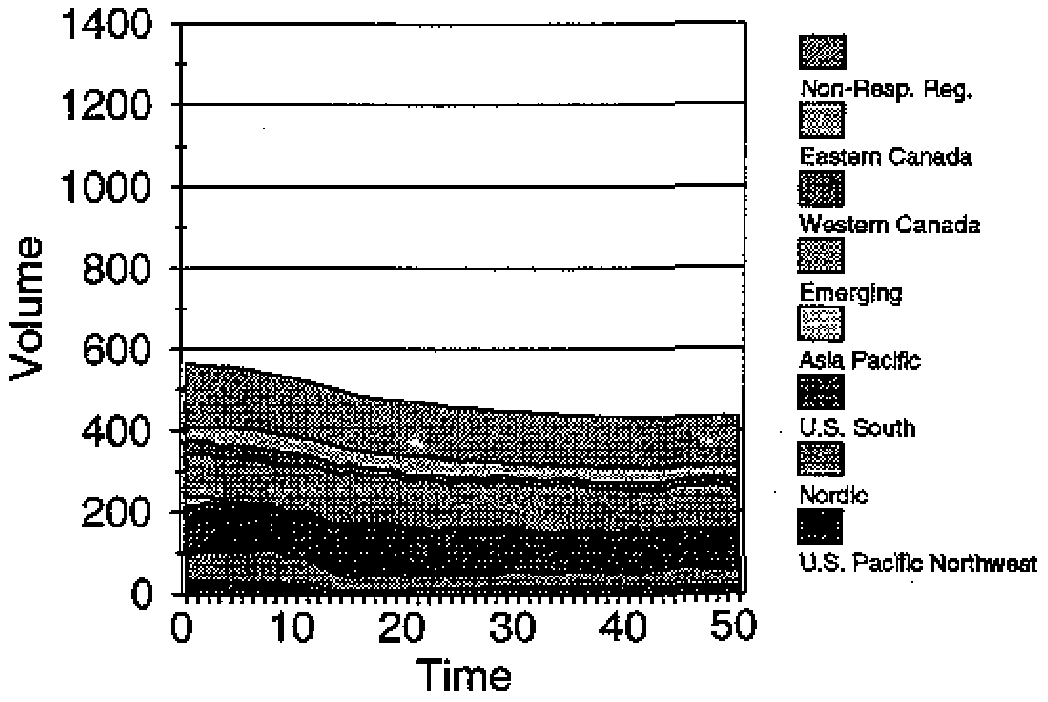
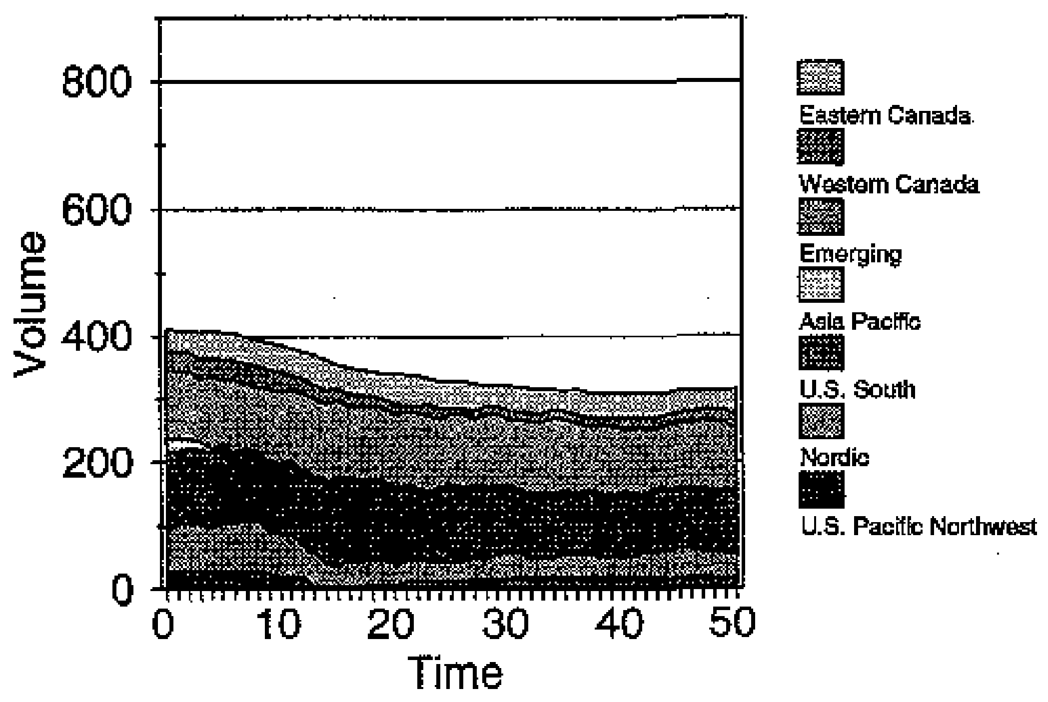


Fig 5.2 Pulpwood: Integ Constr



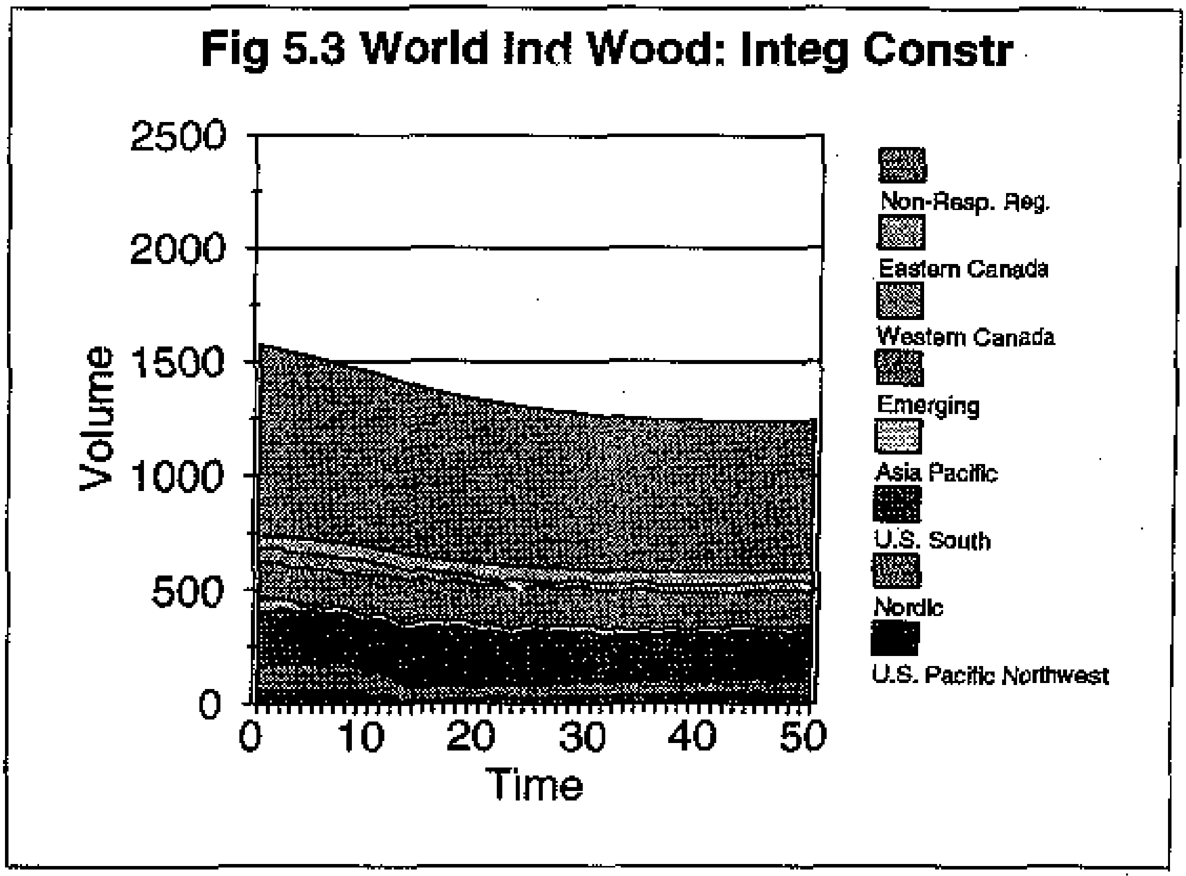


Fig 5.3 World Ind Wood: Integ Constr
Volume
2500
2000
1500
1000
500
0
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Fig 5.4 World Solidwood: Integ Constr

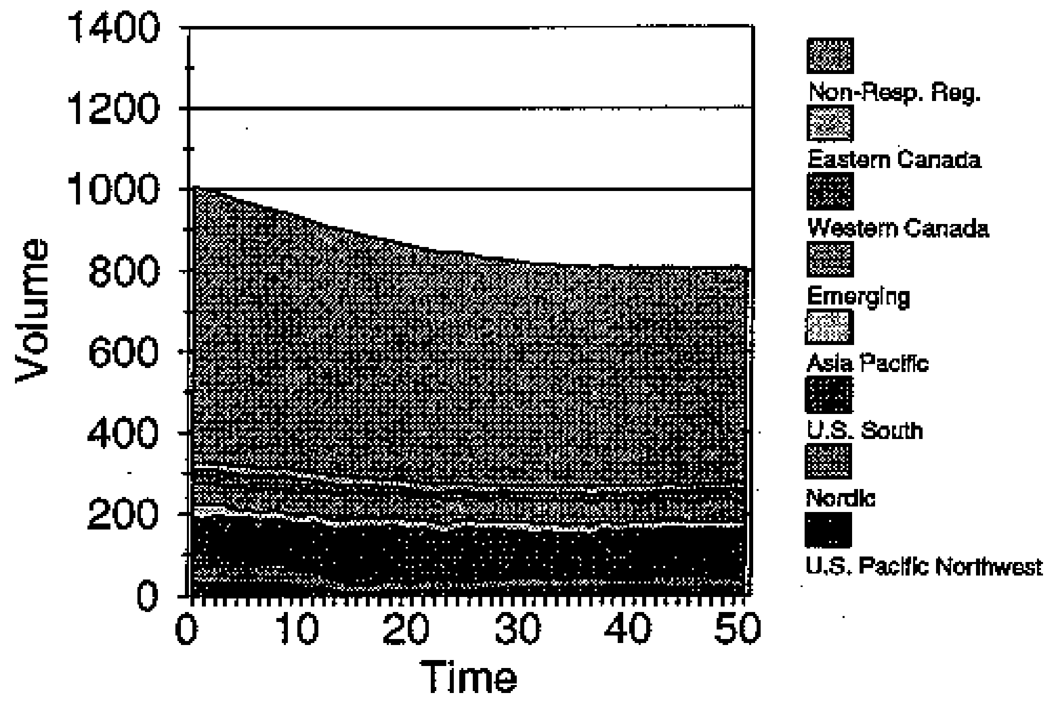


Fig 5.5 Prices:Integrated Constraints

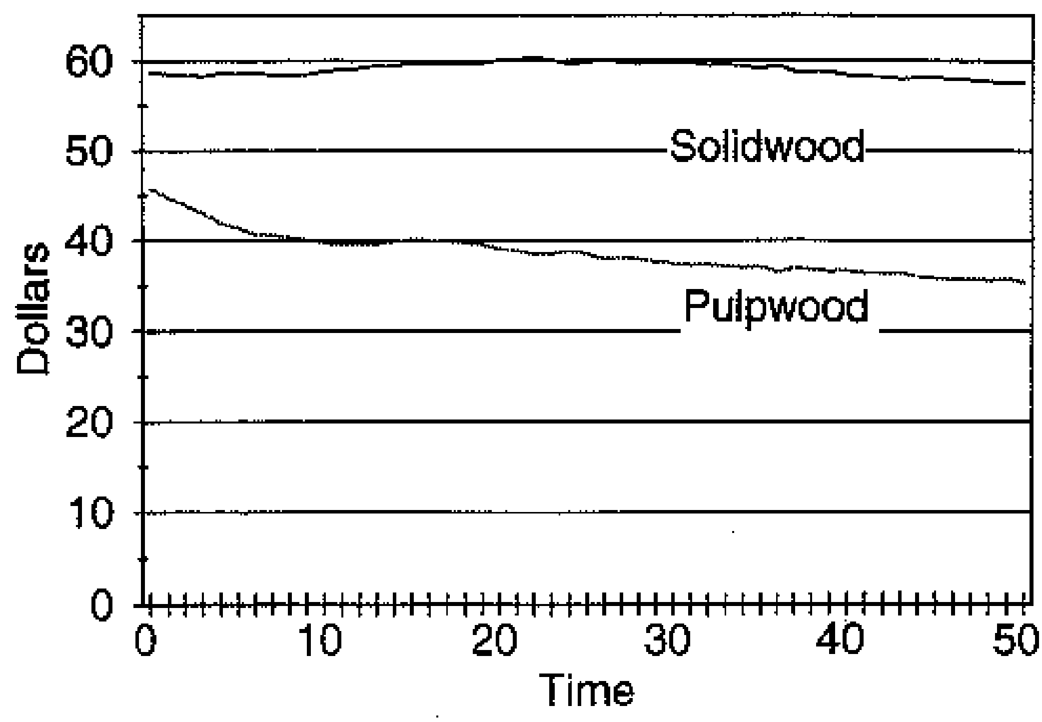


Fig 6.1 Wrld Pulpwood: Integ Con, BC

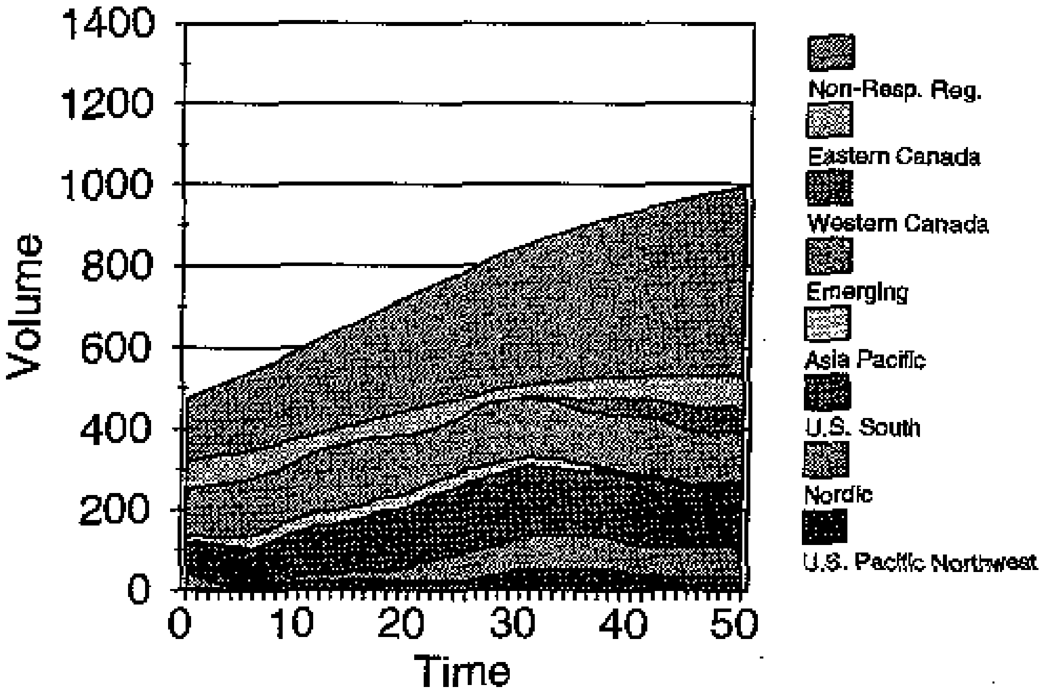
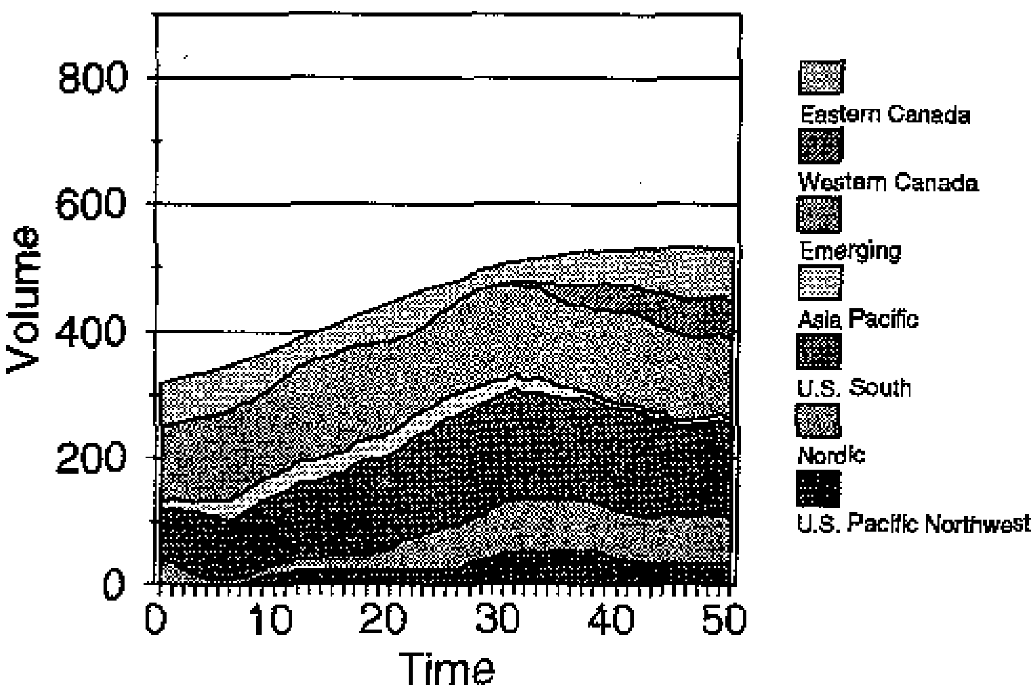


Fig 6.2 Pulpwood , Integ Constr, BC



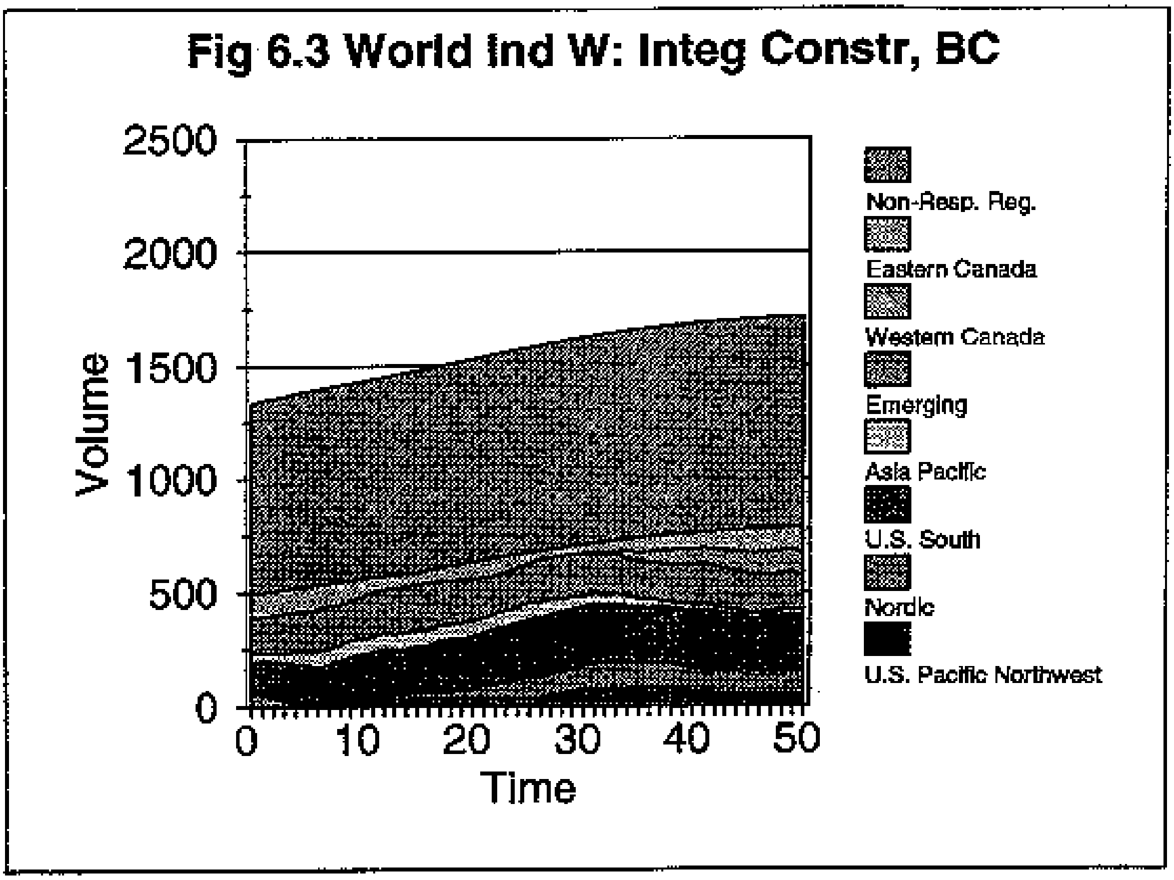


Fig 6.3 World Ind W: Integ Constr, BC
2500
2000
1500
1000
500
0
Volume
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Fig 6.4 Wrld Solidw: Integ Constr, BC

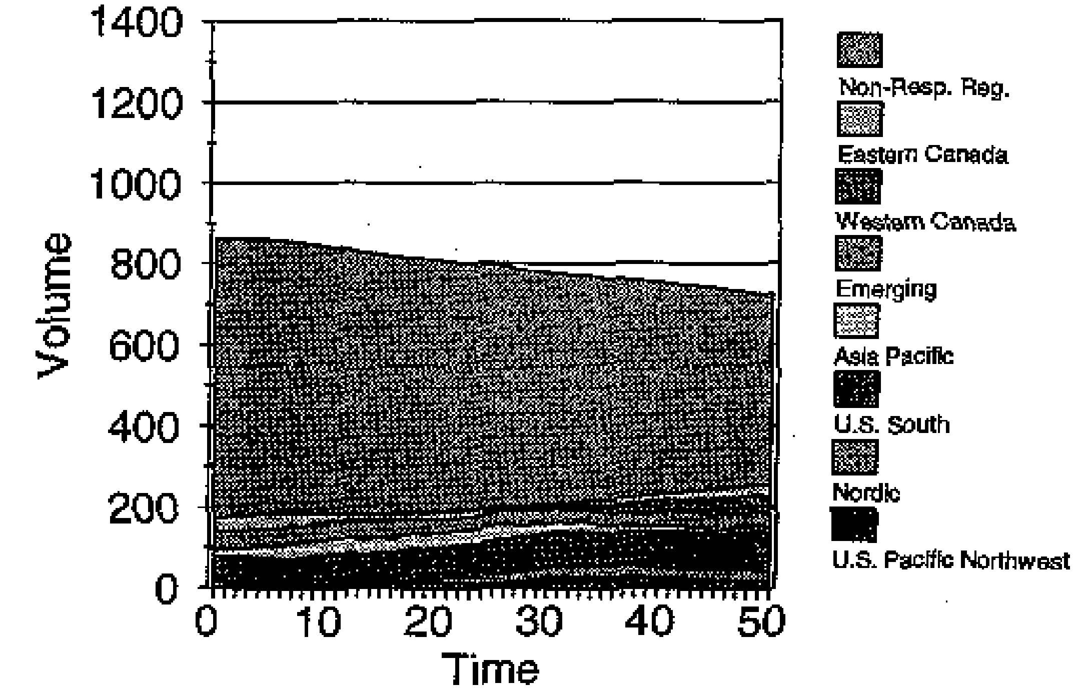
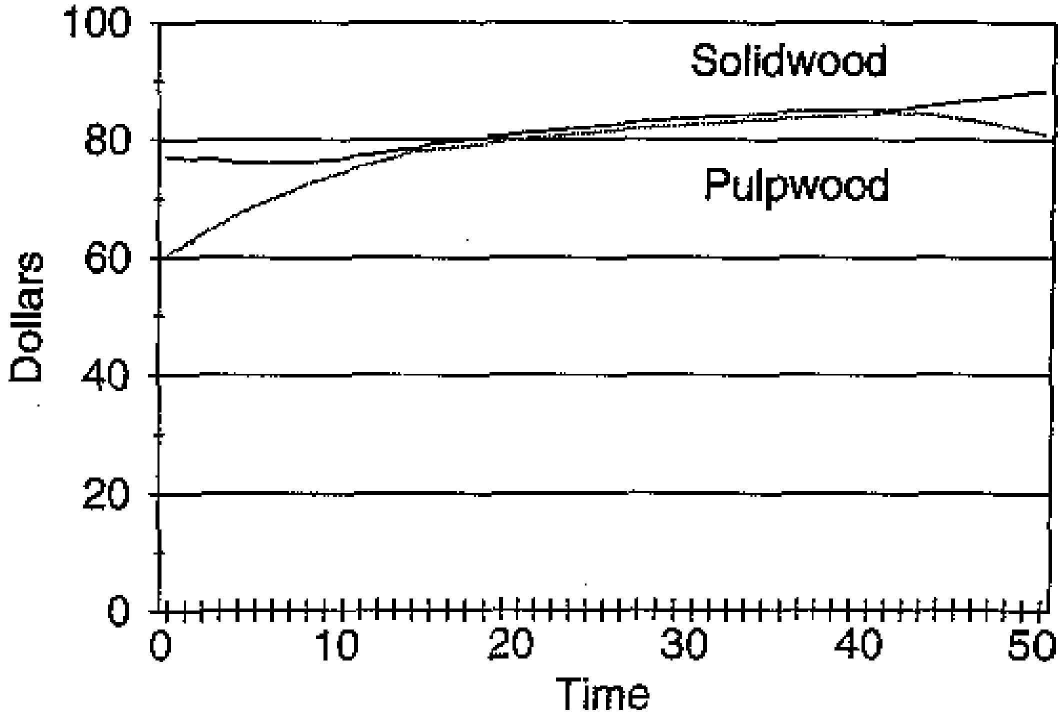


Fig 6.5 Prices: Integ Constraints, BC



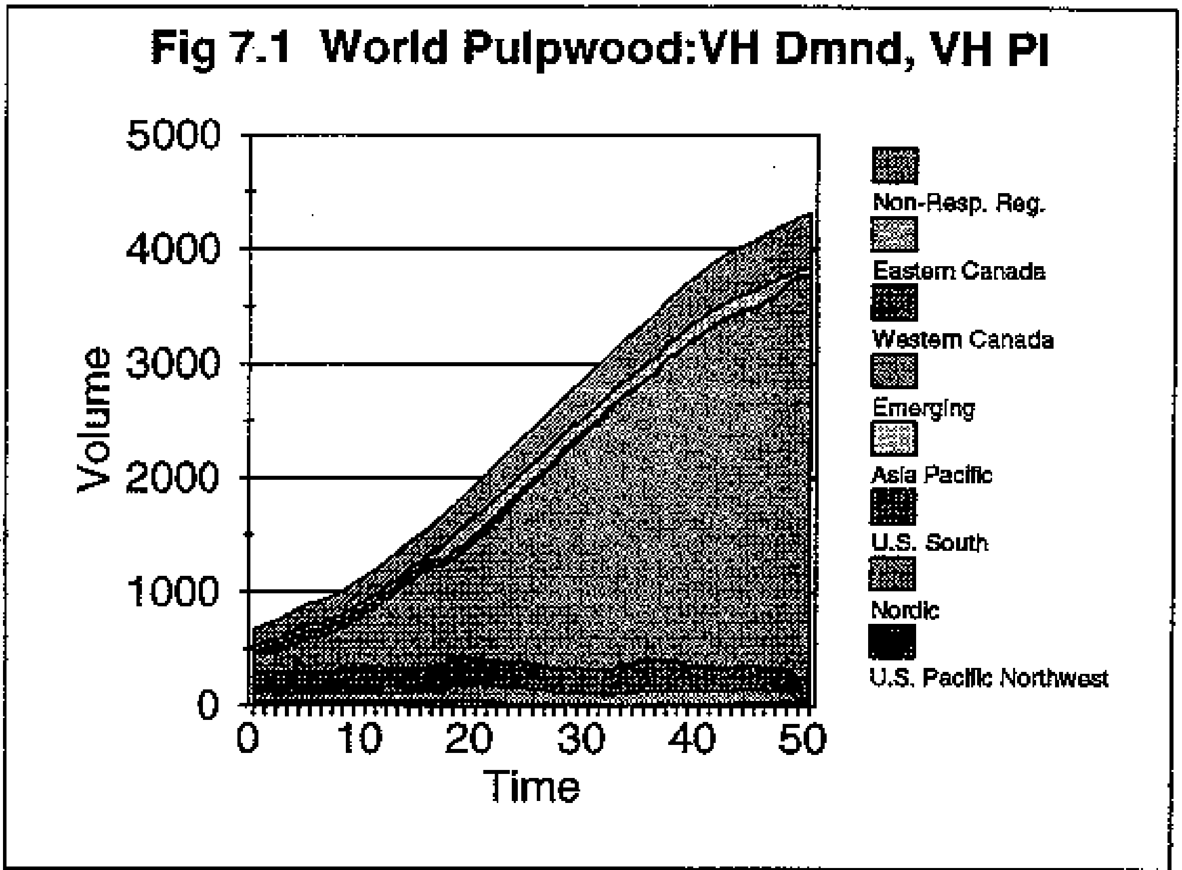


Fig 7.1 World Pulpwood:VH Dmnd, VH PI
5000
4000
3000
2000
1000
0
Volume
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

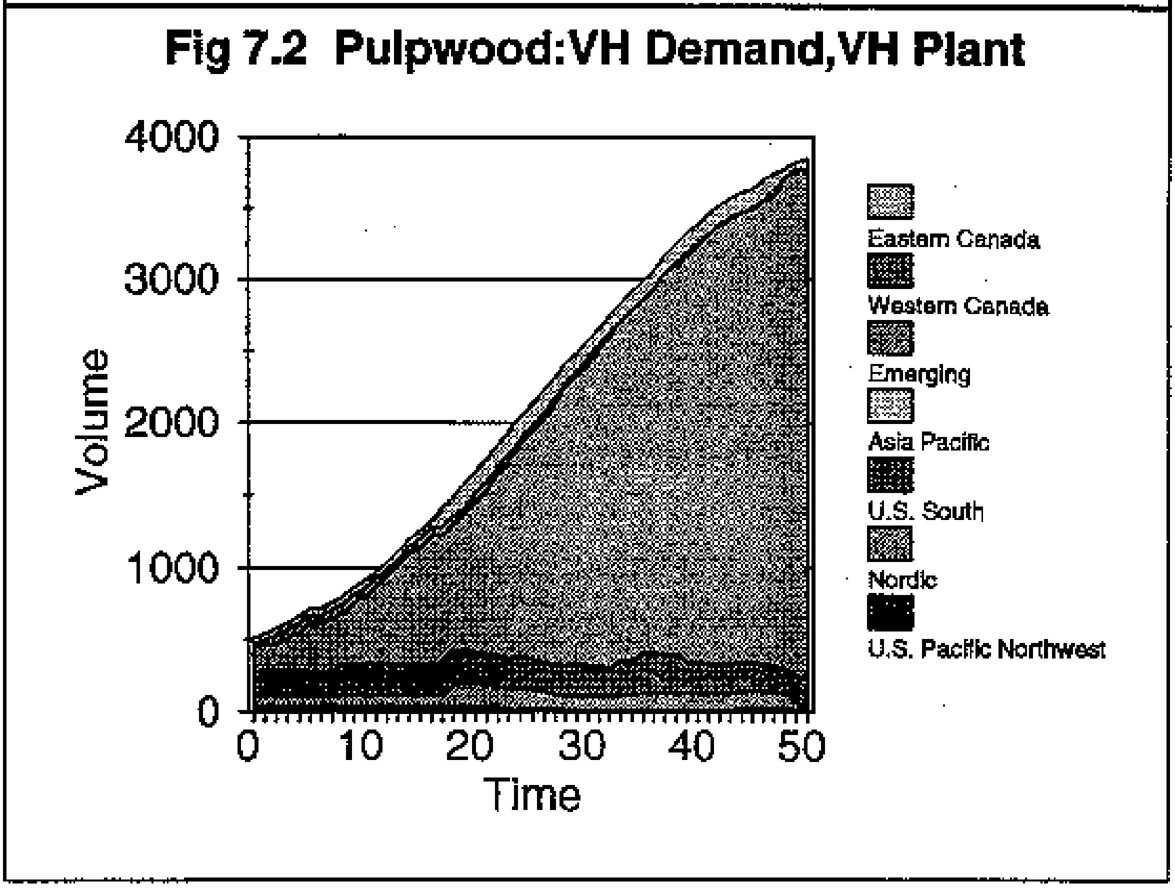


Fig 7.2 Pulpwood:VH Demand,VH Plant
4000
3000
2000
1000
0
Volume
0
10
20
30
40
50
Time
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

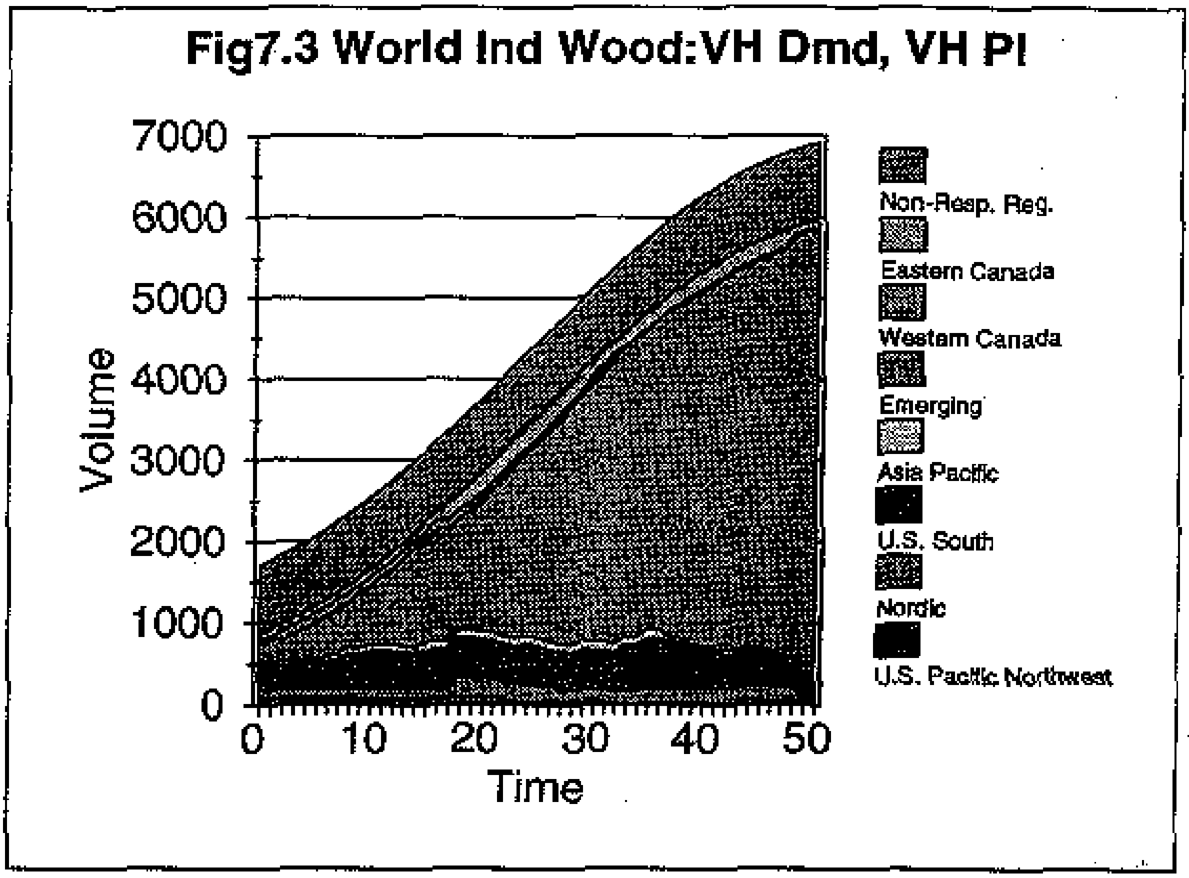


Fig7.3 World Ind Wood:VH Dmd, VH PI
7000
6000
5000
4000
3000
2000
1000
0
Volume
0
10
20
30
40
50
Time
Non-Resp. Reg.
Eastern Canada
Western Canada
Emerging
Asia Pacific
U.S. South
Nordic
U.S. Pacific Northwest

Fig 7.4 World Solidwood:VH Dmd, VH Pl

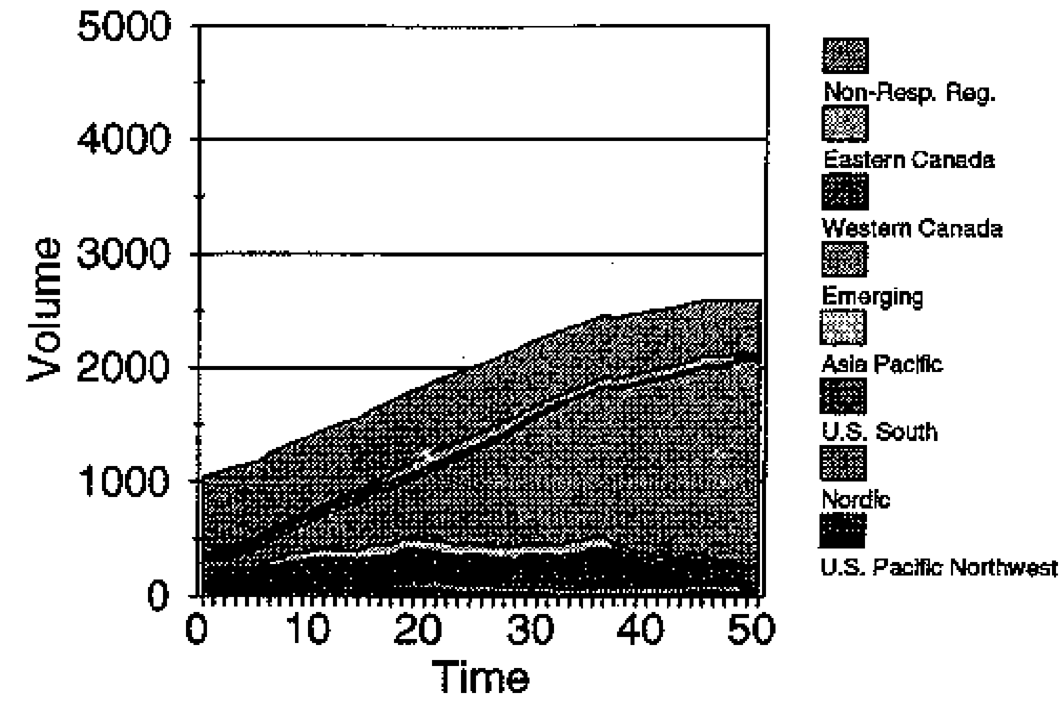
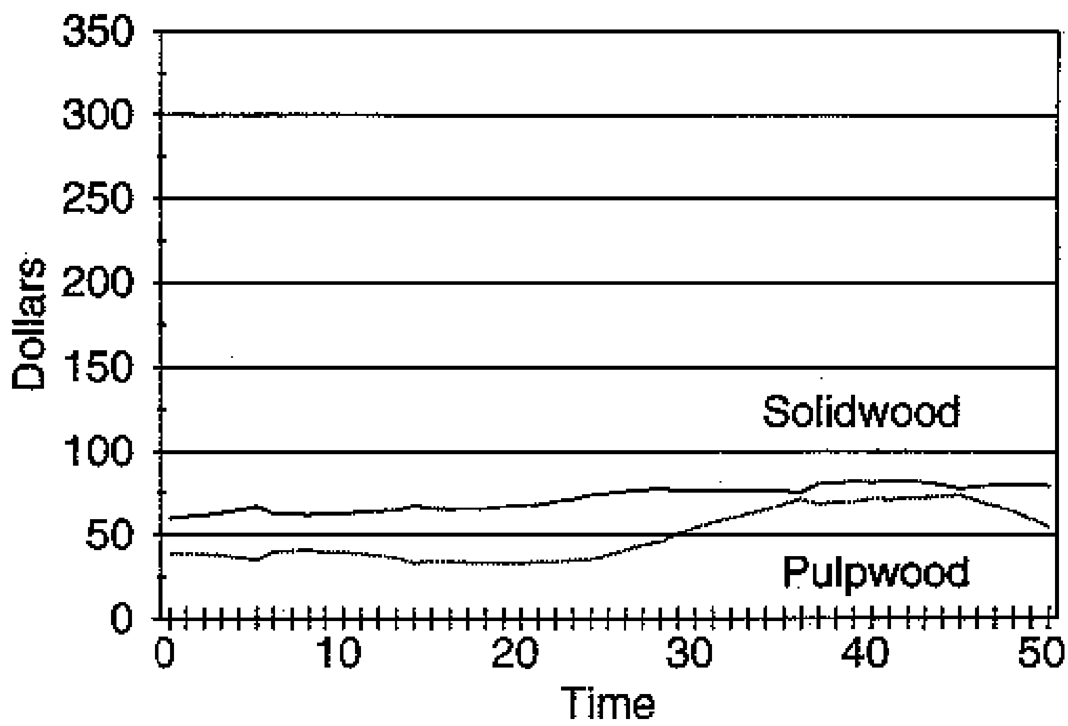


Fig 7.5 Prices:VH Demand, VH Plant



APPENDIX A
TIMBER SUPPLY MODEL WITH SOLIDWOOD AND PULPWOOD

Description of the Model

The model is an extension of our Timber Supply Model presented in *The Long-Term Adequacy of World Timber Supply* (1990). The objective function for the model is the discounted present value of the time stream of net surplus (consumers' plus producers') resulting from the harvests, where in some land classes the harvests are of both solidwood and pulpwood. This function is maximized subject to the initial conditions and the rules governing the evolution (laws of motion) of the system.

We include demand curves for both solidwood and pulpwood, and define net surplus in year j as the area under the year j's demand curve for solidwood from zero to the volume of solidwood harvested plus the area under the year j's demand curve for pulpwood from zero to the volume of pulpwood harvested minus harvesting, transporting, and regeneration costs for year j, where the costs are the sum of costs over the 22 land classes. The volume of merchantable timber is divided between solidwood and pulpwood using variable proportions which are determined endogenously as a function of the price of solidwood relative to the price of pulpwood. These ratios also vary by land class. In addition, each land class has its own cost function which is determined by specific features such as harvesting terrain, log size, accessing terrain, distance to the mill and to the international market. The costs for a land class in year j will depend upon the cost function, the volume harvested, and the hectares of land harvested and regenerated in the land class in year j.

The volume harvested for each land class depends upon the yield function for the land class and the hectares of trees harvested from each age group . The yield function for a land class is determined by characteristics such as climate, terrain, and soil quality, with the yield per hectare a function of the age of trees (years since the trees were regenerated) and the intensity of management practices used on that hectare during this rotation.

The initial conditions are the relevant items from history and they include such items as hectares of forest by age group and land class. They also include the level of the composite regeneration input for each of the hectares. This composite input is the present value of all planting and silvicultural operations including precommercial thinning. We refer to this as the level of the regeneration input or as the intensity of forest management.

The laws of motion for the system are the rules that govern the system. These include the rules for ageing and regenerating the forest. They redefine hectares of trees in age group i in time period j to be hectares of trees in age group $i+1$ in time period $j+1$, and regenerate each harvested hectare in the time period that it is harvested. To simplify the discussion we call age group i trees i years old. Note, however, that this may not be descriptively accurate because of regeneration lag which depends upon the land class involved and the level of the regeneration input applied to these hectares. The rules also redefine the level of the regeneration input associated with age group i in time period j as the level associated with age group $i+1$ in time period $j+1$ in order to keep each hectare of trees and their associated level of the regeneration input together.

The choice, control, variables are for each year the hectares harvested by age group and land class, and the level of the regeneration input for each land class. Selection of these determine the rotation age by land class, the volumes harvested and costs in each year. In addition, with the laws of motion and the initial conditions they determine the structure of the forest, hectares of forest and regeneration input by age group and land class, in each year.

We structure the problem so that it evolves to the stationary state, which is a state where the same flows occur and the same stockes exist each year. This is sometimes called the regulate forest. The computer program of optimization, therefore, first solves for the stationary state solution values of the control and state variables. The state variables are the stock variables discussed above, hectares of trees by age group and land class, and the associated levels of the regeneration input. It solves for the optimal length of the rotation period, the stumpage (net) price of timber, the volume of timber harvested by land class in the stationary state. Then the optimal time profiles of these same variables are calculated for the transition period. This is done by solving the difference equation problem identified by the laws of motion, the first order conditions, the initial conditions, and the terminal conditions, where the terminal conditions are determined by the solution stationary state.

The role of discrete optimal control theory lies in the identification of the laws of motion and the equations and equalities for the necessary conditions. These are used to identify the equations that are iteratively solved to numerically solve the problem.

Formal Model

The net surplus in year j can be written as

$$s_j = \int_0^{Q_j} D_j^f(n)dn + \int_0^{@_j} D_j^p(n)dn - C_j$$

where Q_j is the quantity or volume of timber for solidwood harvested in year j; $D_j^f(Q_j)$ is the inverse form of the demand function for industrial wood for solidwood in year j; $@_j$ is the volume of timber for pulp harvested in year j; $D_j^p(@_j)$ is the demand function for industrial wood for pulp; and C_j is the total cost (expenditures) in year j.

The total costs are the sum of harvest, access, domestic and international transportation cost (CH_j), and regeneration costs (CR_j). Harvesting and transportation costs in year j depend on the total volumes harvested by land class and regeneration expenditures depend upon hectares harvested (regenerated) and level of the input used.

Define x_{hj} to be a vector of hectares of trees by age group for land class h in year j with elements x_{hij}. The subscripts h, i and j are for land class, age group, and year, respectively; thus x_{hij} gives for land class h the number of hectares of age group i trees in year j. Let z_{hj} be the vector of state variables for the regeneration input, with z_{hij} the level of the regeneration input associated with age group i in year j for land class h.

Define u_{hj} to be the control vector of portions of hectares harvested. The elements u_{hij} denote for land class h the portion of the hectares of age group i trees harvested in year j. Let w_{hj} be the level of the regeneration input per hectare for those hectares regenerated in year j, and p_{wh} be the price of the regeneration input for land class h.

The merchantable volume of timber per hectare for land class h in time period j for a stand regenerated i time periods ago depends upon i and upon the magnitude of the regeneration input used on this stand (z_{hij}). We denote this merchantable volume:

$$q_{hij} = f_h(i, z_{hij}) \tag{1}$$

This volume is divided between solidwood and pulpwood using variable proportions which vary by land class, with ϕ_h the portion going to solidwood and $(1 - \phi_h)$ the portion going to pulpwood. The proportion ϕ_h is a constant elasticity function of the price of solidwood relative to the price of pulpwood (p_j^s / p_j^p). When this relative price is greater than or equal to 1.05, it is given by

$$\phi_{hj} = A_h(p_j^s / p_j^p)^\epsilon$$

where p^s and p^p are solidwood and pulpwood price, respectively, ϵ is the elasticity of ϕ with respect to relative price, which is the same for all land classes, and A_h is a scaling factor that varies by land class. When, however, the relative price is between 1 and 1.05 we use the function

$$\phi_{hj} = [(p_j^s / p_j^p) - 1]^{\epsilon 2h}$$

where $\epsilon 2h$ is selected so that these two functions for ϕ_{hj} give the same value at a relative price of 1.05. Note that the value of this function approaches zero as relative price approaches one. For the base case and scenarios we used an elasticity, ϵ, of 0.6 and selected the scaling factors so that the reference percents solid given in Appendix D would exist at a relative price of 1.5.

With these definitions, the volume of commercial timber harvested for solidwood and pulpwood from land class h in year j, Q_{hj}, and $\mathcal{Q}_{hj}$ are given by

$$Q_{hj} = \phi_h u_{hj}' X_{hj} q_{hj} \tag{2a}$$

$$\mathcal{Q}_{hj} = (1 - \phi_h) u_{hj}' X_{hj} q_{hj} \tag{2b}$$

and

$$Q_j = \sum_h Q_{hj}\,, \qquad \mathcal{Q}_j = \sum_h \mathcal{Q}_{hj}$$

where X_{hj} is a diagonal matrix using the elements of x_{hj}, and the total volume harvested in the responsive regions, those that we are modeling, is the sum of these over all land classes.

Harvest, access, and transportation costs for land class h are a function of the volume harvested in that land class

$$CH_{hj} = c_h(Q_{hj} + \mathcal{Q}_{hj}) \tag{3}$$

and regeneration cost for land class h in time period j is given by

$$CR_{hj} = (u'_{hj}x_{hj} + v_{hj})p_{wj}w_{hj} \tag{4}$$

where the inner product in parentheses gives the hectares harvested in land class h, v_{hj} is the exogenously determined number of hectares of new forest land in land class h (v_{hj} is nonzero only in the Emerging Region), and the product of last two terms gives expenditure per hectare. This yields total costs as

$$C_j = \sum_h (CH_{hj} + CR_{hj}) \ .$$

We write the objective function, the present value of the net surplus stream, as:

$$S_0(x_0, z_0, u, w) = s_0 + \rho s_1 + \cdots + \rho^{J-1}s_{J-1} + \rho^J S_J^*(x_J, z_J) \tag{5}$$

where ρ is the discount factor, exp($-r$), with r the real rate of interest, u is any admissible set of control vectors, $u_0, u_1, \ldots, u_{J-1}$ (including all land classes), w is any set of admissible control scalars $w_0, w_1, \ldots, w_{J-1}$ (also covering all land classes), and $S_J^*()$ the optimal terminal value function.

Equation (5) is maximized subject to the laws of motion and the constraints on the values of the control variables. The portions of hectares harvested are constrained to be nonnegative and less than or equal to one, and the regeneration inputs are constrained to be nonnegative:

$$0 \leq u_{hij} \leq 1 \qquad \text{for all } h, i, j \tag{6a}$$

$$0 \leq w_{hj} \qquad \text{for all } h, j \tag{6b}$$

The laws of motion for the system are given by:

$$x_{h,j+1} = (A + BU_{hj})x_{hj} + v_{hj}\, e \qquad \text{for all } h, j \tag{7a}$$

$$z_{h,j+1} = Az_{hj} + w_{hj}\, e \qquad \text{for all } h, j \tag{7b}$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \\ -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & -1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & -1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

A, B, and *U* are *M*-square matrices U_{hj} is a diagonal matrix using the elements of u_{hj}, and e is an M-vector, where M is equal to or greater than the index number of the oldest age group in the problem.

The products Ax_{hj} moves x_{hij} to $x_{h,i+1,j+1}$. Each year each age group becomes older by one year. The product $BU_{hj}x_{hj}$ subtracts from the redefined quantities the hectares harvested and places them in the one-year-old category (newly regenerated category). v_{hj} is the exogenously determined hectares of new forest plantation in time period *j*, and $v_{hj}e$ adds these hectares to the one-year-old age group in time period *j+1*. These can be expressed as:

$$x_{h,1,j+1} = u_{hj}' x_{hj} + v_{hj} \qquad \text{for all } h, j \qquad (8a)$$

$$x_{h,i+1,j+1} = x_{hij} - u_{hij}\, x_{hij} \qquad (i = 1,2,\ldots,M\text{-}1) \qquad (8b)$$

In the law of motion for *z*, the product Az_j moves z_{hij} to $z_{h,i+1,j+1}$. This parallel redefining of *x* and *z* keeps the regenerated hectares and the level of their regeneration input in the same relative position in their respective state vectors. The scalar product $w_{hj}e$ places w_{hj} in location $z_{h1,j+1}$. Thus,

$$z_{h1,j+1} = w_{hj} \qquad \text{for all } h, j \qquad (9a)$$

$$z_{h,i+1,j+1} = z_{hij} \qquad (i = 1,2,\ldots,M\text{-}1) \qquad (9b)$$

<u>Necessary Conditions</u>

The discrete time optimal control literature contains extensive general discussions of the discrete time maximum principle (Abadie 1970, Butkovskii 1963, Dyer and McReynolds 1970, Jackson and Horn 1964, Katz, 1962, and Polak 1971). In this section the maximum principle for this forestry problem is stated and the necessary conditions examined. In the next section this information is linked to the shooting (binary search) method which is used in the solution algorithm. This procedure identifies the margins on which the equations hold and also the specific difference equations that are used to solve the problem numerically.

The maximum principle is a theorem which states that the constrained maximization of equation (5) can be decomposed into a series of subproblems. In each time period, the following Hamiltonian is maximized[1] with respect to u_{hj} and w_{hj} subject to the constraints (equations 6a, 6b, 7a, 7b).

The Hamiltonian for year j is

$$H_j = \int_0^{\alpha_j} D_j^f(n)dn + \int_0^{\omega_j} D_j^p(n)dn - C_j + \sum_h \lambda'_{h,j+1}\,[(A + BU_{hj})x_{hj} + v_{hj}e] + \\ + \sum_h \psi_{h,j+1}\,(Az_{hj} + p_{wh}w_{hj}e) \qquad (10)$$

where

$$\lambda_{hj} = \rho[dS_j^*(x_j, z_j)/dx_{hj}] \qquad (j = 1,\ldots,J)$$

$$\lambda_{hj} = \rho[(ds_j^*/dx_{hj}) + (A + BU_{hj}^*)'\lambda_{h,j+1}] \qquad (j = 1,\ldots,J\text{-}1) \qquad (11a)$$

In addition,

$$\psi_{hj} = \rho[dS_j^*(x_j, z_j)/dz_{hj}] \qquad (j = 1,\ldots,J)$$

$$\psi_{hj} = \rho[(ds_j^*/dz_{hj}) + A'\psi_{h,j+1}] \qquad (j = 1,\ldots,J\text{-}1) \qquad (11b)$$

The derivatives with respect to vectors are gradient vectors, and $S_{j+1}^*(\,)$ is the solution function in $j+1$. The solution function in year $j+1$ can be conceptualized as the result of an application of Bellman's optimality principle and backwards recursion. The λ_{hj} and ψ_{hj} are costate (adjoint) vectors, and identify the shadow values of the hectares of forest and the regeneration input in each age group, respectively, in year j

The necessary conditions for the constrained maximization of the Hamiltonians in equation (10) are both necessary and sufficient for the constrained maximization of equation (5). The correspondence of necessary conditions is the essence of the maximum principle (Halkin 1966). The conditions are sufficient because an equivalent form of the constrained

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[1]In general, the necessary conditions require only that the Hamiltonian be stationary (Jackson and Horn 1965, p.390); however, a stationary value will be a maximum subject to the constraints because the constraints are linear and the Hamiltonian is quasi-concave at a stationary point.

maximization of equation (5) can be shown to be the maximization of a quasi-concave function subject to a set of linear constraints.

The Lagrangean function and the Kuhn-Tucker conditions of this optimization problem are

$$L_j^H = H_j + \sum_h \zeta_{hj}' (1 - u_{hj}) \tag{12}$$

$$dL_j^H/du_{hj} = [\phi_h D_j'(Q_j) + (1 - \phi_h) D_j^p(\mathcal{Q}_j) - c_h'(Q_{hj} + \mathcal{Q}_{hj})] X_{hj} q_{hj} - x_{hj} p_{wh} w_{hj}$$

$$+ X_{hj}' B' \lambda_{h,j+1} - \zeta_{hj} \leq 0 \quad \text{(for all } h\text{)} \tag{13a}$$

$$(\partial L_j^H / \partial u_{hij}) u_{hij} = 0 \quad \text{(for all } h \text{ and } i\text{)} \tag{13b}$$

$$\partial L_j^H / \partial w_{hj} = -u_{hj}' x_{hj} p_{wh} + \psi_{h,1,j+1} \leq 0 \quad \text{(for all } h\text{)} \tag{13c}$$

$$\partial L_j^H / \partial w_{hj}) w_{hj} = 0 \quad \text{(for all } h\text{)} \tag{13d}$$

$$dL_j^H / d\zeta_{hj} = (1 - u_{hj}) \geq 0 \quad \text{(for all } h\text{)} \tag{13e}$$

$$\partial L_j^H / \partial \zeta_{hj}) \zeta_{hj} = 0 \quad \text{(for all } h \text{ and } i\text{)} \tag{13f}$$

These Kuhn-Tucker conditions, the laws of motion for the state variables (equations 7a and 7b), and the laws of motion for the costate variables (equations 11a and 11b) identify a two-point boundary value problem that can be used to solve both theoretical and numerical problems.

Analytic Solution

Equations (7a) and (7b) identify the method for calculating the values of state variables (hectares of forest by age group and stock of regeneration investment) in each year, given the values for the control variables in each year. These equations move the state variables forward through time.

Equations (11a) and (11b) identify the method for calculating the costate variables (shadow values of the state variables) in each year, given the values of the control variables in each year. This procedure calculates the costate variables starting at year J and moving backward through time to the present (year $j = 0$). Finally, equations (13a) through (13f) identify the method of finding the values of control variables in each year.

We manipulate equations (13a) and (13b) into a set of difference equations and use the difference equations in our solution algorithm and in our theoretical discussions of the solution time profiles. The elements of dL_j^H/du_{hj} in equation (13a) can be written:

$$p_{hj} x_{hij} q_{hij} - x_{hij} p_{wh} w_{hj} + x_{hij}(\lambda_{h,1,j+1} - \lambda_{h,i+1,j+1}) - \zeta_{hij} \leq 0 \quad \text{(for all } h \text{ and } i\text{)} \tag{14}$$

where p_{hj} is the net price or stumpage price of timber for land class h. Stumpage price is equal to the market price of solidwood weighted by the portion of timber in land class h that is solidwood, $\phi_h D_j^s(Q_j)$, plus the price of pulpwood weighted by the portion that is pulpwood, $(1 - \phi_h)D_j^p(\mathcal{Q})$, minus the marginal harvesting, accessing, and transportation cost of timber, $c_h'(Q_{hj} + \mathcal{Q}_{hj})$. This equation gives the marginal net surplus (net shadow value) of harvesting hectares of trees by age group and land class.

In equation (14) $\lambda_{h,1,j+1}$ is the shadow value of trees that are one year old in year $j+1$, i.e., trees regenerated in year j. Examination of equation (11a) indicates that its solution value is the discounted value of the actual harvest of these trees in the future. The costate variable $\lambda_{h,i+1,j+1}$ is the discounted value of age group i in land class h from next year. For age group k in this land class it can be written

$$\lambda_{h,k+1,j+1} = \rho(\lambda_{h,1,j+1} + p_{h,j+1}\, q_{h,k+1} - p_{wh}\, w_{h,j+1}) \tag{15}$$

which states that the opportunity cost of harvesting k year old trees in year j is the discounted value of the trees that could be regenerated a year in the future plus the stumpage price of timber next year times the merchantable volume of timber on that hectare one year in the future minus the optimal regeneration expenditures on that hectare.

Combining this last equation and equation (14) yields

$$p_{hj}\, q_{hk} - p_{wh}(w_{hj} - \rho w_{h,j+1}) + \lambda_{h,1,j+1} - \rho\lambda_{h,1,j+2} - \zeta_{hkj} / x_{hkj} - \rho p_{h,j+1} q_{h,k+1} \le 0 \tag{16}$$

The solution to this discrete time optimal control model is the time paths of state variables, control variables, and costate variables, such that the laws of motion for state variables, equations (7a) and (7b), the laws of motion of costate variables, equations (11a) and (11b), the difference equation for the net price of timber (stumpage price) given in equation (16), and the remaining first order condition, equations (13b) through (13f), are simultaneously satisfied. These form a two-point boundary value difference equation problem.

Solution Algorithm

Our solution algorithm solves for the optimal values of the control (choice) variables in the transition period. These time paths generate an evolution of the state variables, e.g. hectares of trees by age group, from their initial values to those in the stationary state. The algorithm uses a shooting (binary search) method to solve a constrained difference equation problem. The difference equations are the laws of motions of the state variables, equations (7a) and (7b); the laws of motions of costate variables, equations (2b) and (6b); and equation (16) which is derived from the first two necessary conditions, equations (13a) and (13b). These form a two-point boundary value difference equation problem, which is to be solved subject to the remaining first order conditions, equation (13c) through (13f). The boundary values are determined by the initial and terminal conditions.

These difference equations with their initial conditions have a set of solutions, one for each value of market price of timber in the first time period. The shooting method is a search for the member of this set that satisfies the terminal conditions. This search is carried out by starting with an arbitrary element from the set of solutions and systematically eliminating solutions that do not satisfy the terminal conditions until a satisfactory "solution" is found.

REFERENCES

Abadie, J. 1970. Application of the GRG algorithm to optimal control problems. Pages 191-211 in J. Abadie, ed. Integer and Nonlinear Programming. North Holland Publishing Company.

Butovskii, A. G. 1963. The necessary and sufficient conditions for optimality of discrete control system. Automatic and Remote Control 24:963-970.

Dyer, P. and D. R. McReynolds. 1970. The Computation and Theory of Optimal Control. Academic Press, New York. 242 p.

Halkin, H. 1966. A maximum principle of the Pontryagin type for systems described by nonlinear difference equations. J. of SIAM Control 4:90-111.

Jackson, R. and F. Horn. 1965. Discrete analogues of Pontryagin's maximum principle. International J. of Control 1:389-395.

Katz, S. 1962. A discrete version of Pontryagin's Maximum Principle. J. of Electronics and Control 11:170-184.

Polak, E. 1971. Computational Methods in Optimization. Academic Press, New York. 329 p.

Sedjo, R.A. and K.S. Lyon. 1990. The Long-Term Adequacy of World Timber Supply, Resources for the Future, Washington D.C. 230 p.

APPENDIX B
SOLIDWOOD/PULPWOOD BY LAND CLASS

Land Class	Percent Solid: Reference	Percent Solid: Year 50 of Base Scenario	Percent Solid: Year 50 of VHD Scenario
1	30	27.1	1.1
2	55	49.6	7.5
3	55	49.6	7.5
4	55	49.6	7.5
5	55	49.6	7.5
6	55	49.6	7.5
7	55	49.6	7.5
8	40	36.1	2.7
9	30	27.1	1.1
10	60	54.2	9.9
11	60	54.2	9.9
12	50	45.1	5.5
13	50	45.1	5.5
14	45	40.6	4.0
15	45	40.6	4.0
16	40	36.1	2.7
17	40	36.1	2.7
18	30	27.1	1.1
19	50	45.1	5.5
20	30	27.1	1.1
21	50	45.1	5.5
22	80	72.2	24.8